

Are we at the dawn of quantum-gravity phenomenology?¹

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ABSTRACT

A handful of recent papers has been devoted to proposals of experiments capable of testing some candidate quantum-gravity phenomena. These lecture notes emphasize those aspects that are most relevant to the questions that inevitably come to mind when one is exposed for the first time to these research developments: How come theory and experiments are finally meeting in spite of all the gloomy forecasts that pervade traditional quantum-gravity reviews? Is this a case of theorists having put forward more and more speculative ideas until a point was reached at which conventional experiments could rule out the proposed phenomena? Or has there been such a remarkable improvement in experimental techniques and ideas that we are now capable of testing plausible candidate quantum-gravity phenomena? These questions are analysed rather carefully for the recent proposals of tests of space-time fuzziness using modern interferometers and tests of dispersion in the quantum-gravity vacuum using observations of electromagnetic radiation from distant astrophysical sources. I also briefly discuss other proposed quantum-gravity experiments, including those exploiting the properties of the neutral-kaon system for tests of quantum-gravity-induced decoherence and those using particle-physics accelerators for tests of models with large extra dimensions. The emerging picture of “quantum-gravity phenomenology” suggests that we are finally starting the exploration of a relatively large class of plausible quantum-gravity effects. However, our chances to obtain positive (discovery) experimental results still depend crucially on the magnitude of these effects; in particular, in most cases the level of sensitivity that the relevant experiments should achieve within a few years corresponds to effects suppressed only linearly by the Planck length.

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1 INTRODUCTION

Traditionally the lack of experimental input [1] has been the most important obstacle in the search for “quantum gravity”, the new theory that should provide a unified description of gravitation and quantum mechanics. Recently there has been a small, but nonetheless encouraging, number of proposals [2, 3, 4, 5, 6, 7, 8, 9] of experiments probing the nature of the interplay between gravitation and quantum mechanics. At the same time the “COW-type” experiments on quantum mechanics in a strong (classical) gravitational environment, initiated by Colella, Overhauser and Werner [10], have reached levels of sensitivity [11] such that even gravitationally induced quantum phases due to local tides can be detected. In light of these developments there is now growing (although still understandably cautious) hope for data-driven insight into the structure of quantum gravity.

The primary objective of these lecture notes is the one of giving the reader an intuitive idea of how far quantum-gravity phenomenology has come. This is somewhat tricky. Traditionally experimental tests of quantum gravity were believed to be not better than a dream. The fact that now (some) theory and (some) experiments finally “meet” could have two very different explanations: it could be that experimental techniques and ideas have improved so much that now tests of plausible quantum-gravity effects are within reach, but it could also be that theorists have managed to come up with scenarios speculative enough to allow testing by conventional experimental techniques. I shall argue that experiments have indeed progressed to the point where some significant quantum-gravity tests are doable. I shall also clarify in which sense the traditional pessimism concerning quantum-gravity experiments was built upon the analysis of a very limited set of experimental ideas, with the significant omission of the possibility (which we now find to be within our capabilities) of experiments set up in such a way that very many of the very small quantum-gravity effects are somehow summed together. Some of the theoretical ideas that can be tested experimentally are of course quite speculative (decoherence, space-time fluctuations, large extra dimensions, ...) but this is not so disappointing because it seems reasonable to expect that the new theory should host a large number of new conceptual/structural elements in order to be capable of reconciling the (apparent) incompatibility between gravitation and quantum mechanics. [An example of motivation for very new structures is discussed here in Section 11, which is a “theory addendum” reviewing some of the arguments [12] in support of the idea [13] that the mechanics on which quantum gravity is based might not be exactly the one of ordinary quantum mechanics, since it should accommodate a somewhat different (non-classical) concept of “measuring apparatus” and a somewhat different relationship between “system” and “measuring apparatus”.]

In giving the reader an intuitive idea of how far quantum-gravity phenomenology has come it will be very useful to rely on simple phenomenological models of candidate quantum-gravity effects. The position I am here taking is not that these models should become cornerstones of theoretical work on quantum gravity (at best they are possible ways in which quantum-gravity might manifest itself), but rather that these models can be useful in giving an intuitive description of the level of sensitivity that experiments are finally reaching. Depending on the reader’s intuition for the quantum-gravity realm these phenomenological models might or might not appear likely as faithful descriptions of effects actually present in quantum-gravity, but in any case by the end of these notes the reader should find that these models are at least useful for the characterization of the level of sensitivity that quantum-gravity experiments have reached, and can also be useful to describe the progress (past and future) of these sensitivity levels. In particular, in the “language” set up by these models one can see an emerging picture suggesting that we are finally ready for the exploration of a relatively large class of plausible quantum-gravity effects, even though our chances to obtain positive (discovery) experimental results still depend crucially on the magnitude of these effects: in most cases the level of sensitivity that the relevant experiments should achieve within a few years corresponds to effects suppressed only linearly by the Planck length L_p ($L_p \sim 10^{-35}m$).

The bulk of these notes gives brief reviews of the quantum-gravity experiments that can be done. The reader will be asked to forgive the fact that this review is not very balanced. The two proposals in which this author has been involved [5, 7] are in fact discussed in greater detail, while for the experiments proposed in Refs. [2, 3, 4, 8, 9] I just give a very brief discussion with emphasis on the most important conceptual ingredients.

The students who attended the School might be surprised to find the material presented with a completely different strategy. While my lectures in Polanica were sharply divided in a first part on theory and a second part on experiments, here some of the theoretical intuition is presented while discussing the experiments. It appears to me that this strategy might be better suited for a written presentation. I also thought it might be useful to start with the conclusions, which are given in the next two sections. Section 4 reviews the proposal of using modern interferometers to set bounds on space-time fuzziness. In Section 5 I review the proposal of using data on GRBs (gamma-ray bursts) to investigate possible quantum-gravity induced *in vacuo* dispersion of electromagnetic radiation. In Section 6 I give brief reviews of other quantum-gravity experiments. In Section 7 I give a brief discussion of the mentioned “COW-type” experiments testing quantum mechanics in a strong classical-gravity environment. Section 8 provides a “theory addendum” on various scenarios for bounds on the measurability of distances in quantum gravity and their possible relation to properties of the space-time foam. Section 9 provides a theory addendum on an absolute bound on the measurability of the amplitude of a gravity wave which should hold even if distances are not fuzzy. Section 10 provides a theory addendum on other works which are in one way or another related to (or relevant for) the content of these notes. Section 11 gives the mentioned theory addendum concerning ideas on a mechanics for quantum gravity that be not exactly of the type of ordinary quantum mechanics. Finally in Section 12 I give some comments on the outlook of quantum-gravity phenomenology, and I also emphasize the fact that, whether or not they turn out to be helpful for quantum gravity, most of the experiments considered in these notes are intrinsically significant as tests of quantum mechanics and/or tests of fundamental symmetries.

2 FIRST THE CONCLUSIONS: WHAT HAS THIS PHENOMENOLOGY ACHIEVED?

Let me start with a brief description of the present status of quantum-gravity phenomenology. Some of the points made in this section are supported by analyses which will be reviewed in the following sections. The crucial question is: Can we just test some wildly speculative ideas which have somehow surfaced in the quantum-gravity literature? Or can we test even some plausible candidate quantum-gravity phenomena?

Before answering these questions it is appropriate to comment on the general expectations we have for quantum gravity. It has been realized for some time now that by combining elements of gravity with elements of quantum mechanics one is led to “interplay phenomena” with rather distinctive signatures, such as quantum fluctuations of space-time [14, 15, 16], and violations of Lorentz and/or CPT symmetries [17, 18, 19, 20, 21, 22, 23], but the relevant effects are expected to be very small (because of the smallness of the Planck length). Therefore in this “intuition-building” section the reader must expect from me a description of experiments with a remarkable sensitivity to the new phenomena.

Let me start from the possibility of quantum fluctuations of space-time. A prediction of nearly all approaches to the unification of gravitation and quantum mechanics is that at very short distances the sharp classical concept of space-time should give way to a somewhat “fuzzy” (or “foamy”) picture, possibly involving virulent geometry fluctuations (sometimes depicted as wormholes and black holes popping in and out of the vacuum). Although the idea of space-time foam remains somewhat vague and it appears to have significantly different

incarnations in different quantum-gravity approaches, a plausible expectation that emerges from this framework is that the distance between two bodies “immersed” in the space-time foam would be affected by (quantum) fluctuations. If urged to give a rough description of these fluctuations at present theorists can only guess that they would be of Planck-length magnitude and occurring at a frequency of roughly one per Planck time T_p ($T_p = L_p/c \sim 10^{-44}s$). One should therefore deem significant for space-time-foam research any experiment that monitors the distances between two bodies with enough sensitivity to test this type of fluctuations. This is exactly what was achieved by the analysis reported in Refs. [7, 24], which was based on the observation that the most advanced modern interferometers (the ones normally used for detection of classical gravity waves) are the natural instruments to study the fuzziness of distances. While I postpone to Section 4 a detailed discussion of these interferometry-based tests of fuzziness, let me emphasize already here that modern interferometers have achieved such a level of sensitivity that we are already in a position to **rule out** fluctuations in the distances of their test masses of the type discussed above, *i.e.* fluctuations of Planck-length magnitude occurring at a rate of one per each Planck time. This is perhaps the simplest way for the reader to picture intuitively the type of objectives already reached by quantum-gravity phenomenology.

Another very intuitive measure of the maturity of quantum-gravity phenomenology comes from the studies of *in vacuo* dispersion proposed in Ref. [5] (also see the more recent purely experimental analyses [25, 26]). Deformed dispersion relations are a rather natural possibility for quantum gravity. For example, they emerge naturally in quantum gravity scenarios requiring a modification of Lorentz symmetry. Modifications of Lorentz symmetry could result from space-time discreteness (*e.g.* a discrete space accommodates a somewhat different concept of “rotation” with respect to the one of ordinary continuous spaces), a possibility extensively investigated in the quantum gravity literature (see, *e.g.*, Ref. [22]), and it would also naturally result from an “active” quantum-gravity vacuum of the type advocated by Wheeler and Hawking [14, 15] (such a “vacuum” might physically label the space-time points, rendering possible the selection of a “preferred frame”). The specific structure of the deformation can differ significantly from model to model. Assuming that the deformation admits a series expansion at small energies E , and parametrizing the deformation in terms of an energy³ scale E_{QG} (a scale characterizing the onset of quantum-gravity dispersion effects, often identified with the Planck energy $E_p = \hbar c/L_p \sim 10^{19} GeV$), for a massless particle one would expect to be able to approximate the deformed dispersion relation at low energies according to

$$c^2 \mathbf{p}^2 \simeq E^2 \left[1 + \xi \left(\frac{E}{E_{QG}} \right)^\alpha + O \left(\left(\frac{E}{E_{QG}} \right)^{\alpha+1} \right) \right] \quad (1)$$

where c is the conventional speed-of-light constant. The scale E_{QG} , the power α and the sign ambiguity $\xi = \pm 1$ would be fixed in a given dynamical framework; for example, in some of the approaches based on dimensionful quantum deformations of Poincaré symmetries [21, 27, 28] one encounters a dispersion relation $c^2 \mathbf{p}^2 = E_{QG}^2 [1 - e^{E/E_{QG}}]^2$, which implies $\xi = \alpha = 1$. Because of the smallness of $1/E_{QG}$ it was traditionally believed that this effect could not be seriously tested experimentally (*i.e.* that, for $E_{QG} \sim E_p$, experiments would only be sensitive to values of α much smaller than 1), but in Ref. [5] it was observed that recent progress in the phenomenology of GRBs [29] and other astrophysical phenomena should soon allow us to probe values of E_{QG} of the order of (or even greater than) E_p for values of α as large as 1. As discussed later in these notes, $\alpha = 1$ appears to be the smallest value that can be obtained with plausible quantum-gravity arguments and several of these arguments actually point us toward the larger value $\alpha = 2$, which is still very far from present-day experimental

³I parametrize deformations of dispersion relations in terms of an energy scale E_{QG} , while I later parametrize the proposals for distance fuzziness with a length scale L_{QG} .

capabilities. While of course it would be very important to achieve sensitivity to both the $\alpha = 1$ and the $\alpha = 2$ scenarios, the fact that we will soon test $\alpha = 1$ is a significant first step.

Another recently proposed quantum-gravity experiment concerns possible violations of CPT invariance. This is a rather general prediction of quantum-gravity approaches, which for example can be due to elements of nonlocality (locality is one of the hypotheses of the “CPT theorem”) and/or elements of decoherence present in the approach. At least some level of non-locality is quite natural for quantum gravity as a theory with a natural length scale which might play the role of “minimum length” [30, 31, 32, 12, 33]. Motivated by the structure of “Liouville strings” [19] (a non-critical string approach to quantum gravity which appears to admit a space-time foam picture) a phenomenological parametrization of quantum-gravity induced CPT violation in the neutral-kaon system has been proposed in Refs. [17, 34]. (Other studies of the phenomenology of CPT violation can be found in Ref. [20, 35].) In estimating the parameters that appear in this phenomenological model the crucial point is as usual the overall suppression given by some power of the Planck length $L_p \sim 1/E_p$. For the case in which the Planck length enters only linearly in the relevant formulas, experiments investigating the properties of neutral kaons are already setting significant bounds on the parameters of this phenomenological approach [2].

In summary, experiments are reaching significant sensitivity with respect to all of the frequently discussed features of quantum gravity that I mentioned at the beginning of this section: space-time fuzziness, violations of Lorentz invariance, and violations of CPT invariance. Other quantum-gravity experiments, which I shall discuss later in these notes, can probe other candidate quantum-gravity phenomena, giving additional breadth to quantum-gravity phenomenology.

Before closing this section there is one more answer I should give: how could this happen in spite of all the gloomy forecasts which one finds in most quantum-gravity review papers? The answer is actually simple. Those gloomy forecasts were based on the observation that under ordinary conditions the direct detection of a single quantum-gravity phenomenon would be well beyond our capabilities if the magnitude of the phenomenon is suppressed by the smallness of the Planck length. For example, in particle-physics contexts this is seen in the fact that the contribution from “gravitons” (the conjectured mediators of quantum-gravity interactions) to particle-physics processes with center-of-mass energy \mathcal{E} is expected to be penalized by overall factors given by some power of the ratio $\mathcal{E}/(10^{19}\text{GeV})$. However, small effects can become observable in special contexts and in particular one can always search for an experimental setup such that a very large number of the very small quantum-gravity contributions are effectively summed together. This later possibility is not unknown to the particle-physics community, since it has been exploited in the context of investigations of the particle-physics theories unifying the strong and electroweak interactions, where one encounters the phenomenon of proton decay. By keeping under observation very large numbers of protons, experimentalists have managed⁴ to set highly significant bounds on proton decay [37], even though the proton-decay probability is penalized by the fourth power of the small ratio between the proton mass, which is of order 1GeV , and the mass of the vector bosons expected to mediate proton decay, which is conjectured to be of order 10^{16}GeV . Just like proton-decay experiments are based on a simple way to put together very many of the small proton-decay effects⁵ the experiments using modern interferometers to study space-time fuzziness and the experiments using GRBs to study violations of Lorentz invariance exploit simple ways to put together very many of the very small quantum-gravity effects. I shall explain this in detail in Sections 4 and 5.

⁴This author’s familiarity [36] with the accomplishments of proton-decay experiments has certainly contributed to the moderate optimism for the outlook of quantum-gravity phenomenology which is found in these notes.

⁵For each of the protons being monitored the probability of decay is extremely small, but there is a significantly large probability that at least one of the many monitored protons decay.

3 ADDENDUM TO CONCLUSIONS: ANY HINTS TO THEORISTS FROM EXPERIMENTS?

In the preceding section I have argued that quantum-gravity phenomenology, even being as it is in its infancy, is already starting to provide the first significant tests of plausible candidate quantum-gravity phenomena. It is of course just “scratching the surface” of whatever “volume” contains the full collection of experimental studies we might wish to perform, but we are finally getting started. Of course, a phenomenology programme is meant to provide input to the theorists working in the area, and therefore one measure of the achievements of a phenomenology programme is given by the impact it is having on theory studies. In the case of quantum-gravity experiments the flow of information from experiments to theory will take some time. The primary reason is that most quantum-gravity approaches have been guided (just because there was no alternative guidance from data) by various sorts of formal intuition for quantum gravity (which of course remain pure speculations as long as they are not confirmed by experiments). This is in particular true for the two most popular approaches to the unification of gravitation and quantum mechanics, *i.e.* “critical superstrings” [38, 39] and “canonical/loop quantum gravity” [40]. Because of the type of intuition that went into them, it is not surprising that these “formalism-driven quantum gravity approaches” are proving extremely useful in providing us new ideas on how gravitation and quantum mechanics could resolve the apparent conflicts between their conceptual structures, but they are not giving us any ideas on which experiments could give insight into the nature of quantum gravity. The hope that these approaches could eventually lead to new intuitions for the nature of space-time at very short distances has been realized only rather limitedly. In particular, it is still unclear if and how these formalisms host the mentioned scenarios for quantum fluctuations of space-time and violations of Lorentz and/or CPT symmetries. The nature of the quantum-gravity vacuum (in the sense discussed in the preceding section) appears to be still very far ahead in the critical superstring research programme and its analysis is only at a very preliminary stage within canonical/loop quantum gravity. In order for the experiments discussed in these notes to affect directly critical superstring research and research in canonical/loop quantum gravity it is necessary to make substantial progress in the analysis of the physical implications of these formalisms.

Still, in an indirect way the recent results of quantum-gravity phenomenology have already started to have an impact on theory work in these quantum gravity approaches. The fact that it is becoming clear that (at least a few) quantum-gravity experiments can be done has reenergized efforts to explore the physical implications of the formalisms. The best example of this way in which phenomenology can influence “pure theory” work is provided by Ref. [41], which was motivated by the results reported in Ref. [5] and showed that canonical/loop quantum gravity admits (under certain conditions, which in particular involve some parity breaking) the phenomenon of deformed dispersion relations, with deformation going linearly with the Planck length.

While the impact on theory work in the formalism-driven quantum gravity approaches is still quite limited, of course the new experiments are providing useful input for more intuitive/phenomenological theoretical work on quantum gravity. For example, the analysis reported in Refs. [7, 24], by ruling out the scheme of distance fluctuations of Planck length magnitude occurring at a rate of one per Planck time, has had significant impact [24, 42] on the line of research which has been deriving intuitive pictures of properties of quantum space-time from analyses of measurability and uncertainty relations [12, 43, 44, 45]. Similarly the “Liouville-string” [19] inspired phenomenological approach to quantum gravity [34, 46] has already received important input from the mentioned studies of the neutral-kaon system and will receive equally important input from the mentioned GRB experiments, once these experiments (in a few years) reach Planck-scale sensitivity.

4 INTERFEROMETRY AND FUZZY SPACE-TIME

In the preceding two sections I have described the conclusions which I believe to be supported by the present status of quantum-gravity phenomenology. Let me now start providing some support for those conclusions by reviewing my proposal [7, 24] of using modern interferometers to set bounds on space-time fuzziness. I shall articulate this in subsections because some preliminaries are in order. Before going to the analysis of experimental data it is in fact necessary to give a proper (operative) definition of fuzzy distance and give a description of the type of stochastic properties one might expect of quantum-gravity-induced fluctuations of distances.

4.1 Operative definition of fuzzy distance

While nearly all approaches to the unification of gravity and quantum mechanics appear to lead to a somewhat fuzzy picture of space-time, within the various formalisms it is often difficult to characterize physically this fuzziness. Rather than starting from formalism, I shall advocate an operative definition of fuzzy space-time. More precisely for the time being I shall just consider the concept of fuzzy distance. I shall be guided by the expectation that at very short distances the sharp classical concept of distance should give way to a somewhat fuzzy distance. Since interferometers are ideally suited to monitor the distance between test masses, I choose as operative definition of quantum-gravity-induced fuzziness one which is expressed in terms of quantum-gravity-induced noise in the read-out of interferometers.

In order to properly discuss this proposed definition it will prove useful to briefly review some aspects of the physics of modern Michelson-type interferometers. These are schematically composed [47] of a (laser) light source, a beam splitter and two fully-reflecting mirrors placed at a distance L from the beam splitter in orthogonal directions. The light beam is decomposed by the beam splitter into a transmitted beam directed toward one of the mirrors and a reflected beam directed toward the other mirror; the beams are then reflected by the mirrors back toward the beam splitter, where [47] they are superposed⁶. The resulting interference pattern is extremely sensitive to changes in the positions of the mirrors relative to the beam splitter. The achievable sensitivity is so high that planned interferometers [48, 49] with arm lengths L of 3 or 4 Km expect to detect gravity waves of amplitude h as low as $3 \cdot 10^{-22}$ at frequencies of about $100Hz$. This roughly means that these modern gravity-wave interferometers should monitor the (relative) positions of their test masses (the beam splitter and the mirrors) with an accuracy [50] of order $10^{-18}m$ and better.

In achieving this remarkable accuracy experimentalists must deal with classical-physics displacement noise sources (*e.g.*, thermal and seismic effects induce fluctuations in the relative positions of the test masses) and displacement noise sources associated to effects of ordinary quantum mechanics (*e.g.*, the combined minimization of *photon shot noise* and *radiation pressure noise* leads to an irreducible noise source which has its root in ordinary quantum mechanics [47]). The operative definition of fuzzy distance which I advocate characterizes the corresponding quantum-gravity effects as an additional source of displacement noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that even in the idealized limit in which all classical-physics and ordinary-quantum-mechanics noise sources are completely eliminated the read-out of an interferometer would still be noisy as a result of quantum-gravity effects.

⁶Although all modern interferometers rely on the technique of folded interferometer's arms (the light beam bounces several times between the beam splitter and the mirrors before superposition), I shall just discuss the simpler “no-folding” conceptual setup. The readers familiar with the subject can easily realize that the observations here reported also apply to more realistic setups, although in some steps of the derivations the length L would have to be understood as the optical length (given by the actual length of the arms multiplied by the number of foldings).

Upon adopting this operative definition of fuzzy distance, interferometers are of course the natural tools for experimental tests of proposed distance-fuzziness scenarios.

I am only properly discussing distance fuzziness although ideas on space-time foam would also motivate investigations of time fuzziness. It is not hard to modify the definition here advocated for distance fuzziness to describe time fuzziness by replacing the interferometer with some device that keeps track of the synchronization of a pair of clocks⁷. I shall not pursue this matter further since I seem to understand⁸ that sensitivity to time fluctuations is still significantly behind the type of sensitivity to distance fluctuations achievable with modern Michelson-type experiments.

4.2 Random-walk noise from random-walk models of quantum space-time fluctuations

As already mentioned in Section 2, it is plausible that a quantum space-time might involve fluctuations of magnitude L_p occurring at a rate of roughly one per each time interval of magnitude $t_p = L_p/c \sim 10^{-44}s$. One can start investigating this scenario by considering the possibility that experiments monitoring the distance D between two bodies for a time T_{obs} (in the sense appropriate, *e.g.*, for an interferometer) could involve a total effect amounting to $n_{obs} \equiv T_{obs}/t_p$ randomly directed fluctuations⁹ of magnitude L_p . An elementary analysis allows to establish that in such a context the root-mean-square deviation σ_D would be proportional to $\sqrt{T_{obs}}$:

$$\sigma_D \sim \sqrt{c L_p T_{obs}} . \quad (2)$$

From the type of T_{obs} -dependence of Eq. (2) it follows [7] that the corresponding quantum fluctuations should have displacement amplitude spectral density $S(f)$ with the f^{-1} dependence¹⁰ typical of “random walk noise” [51]:

$$S(f) = f^{-1} \sqrt{c L_p} . \quad (3)$$

In fact, there is a general connection between $\sigma \sim \sqrt{T_{obs}}$ and $S(f) \sim f^{-1}$, which follows [51] from the general relation

$$\sigma^2 = \int_{1/T_{obs}}^{f_{max}} [S(f)]^2 df , \quad (4)$$

⁷Actually, a realistic analysis of ordinary Michelson-type interferometers is likely to lead to a contribution from space-time foam to noise levels that is the sum (in some appropriate sense) of the effects due to distance fuzziness and time fuzziness (*e.g.* associated to the frequency/time measurements involved).

⁸This understanding is mostly based on recent conversations with G. Busca and P. Thomann who are involved in the next generation of ultra-precise clocks to be realized in microgravity (outer space) environments.

⁹One might actually expect even more than T_{obs}/t_p fluctuations of magnitude L_p in a time T_{obs} depending on how frequent fluctuations occur in the region of space spanned by the distance D . This and other possibilities will be later modelled by replacing L_p with a phenomenological scale L_{QG} which could even depend on D . However, as mentioned in the Introduction, rather than focusing on the details of the physics of the fuzziness models, I am here discussing models from the point of view of a characterization of the levels of quantum-gravity sensitivity reached by recent experiments, and the scale L_{QG} will be seen primarily from this perspective rather than attempting careful estimates in terms of one or another picture of space-time fluctuations.

¹⁰Of course, in light of the nature of the arguments used, one expects that an f^{-1} dependence of the quantum-gravity induced $S(f)$ could only be valid for frequencies f significantly smaller than the Planck frequency c/L_p and significantly larger than the inverse of the time scale over which, even ignoring the gravitational field generated by the devices, the classical geometry of the space-time region where the experiment is performed manifests significant curvature effects.

valid for a frequency band limited from below only by the time of observation T_{obs} .

The displacement amplitude spectral density (3) provides a very useful characterization of the random-walk model of quantum space-time fluctuations prescribing fluctuations of magnitude L_p occurring at a rate of roughly one per each time interval of magnitude L_p/c . If somehow we have been assuming the wrong magnitude of distance fluctuations or the wrong rate (also see Subsection 4.4) but we have been correct in taking a random-walk model of quantum space-time fluctuations Eq. (3) should be replaced by

$$S(f) = f^{-1} \sqrt{c L_{QG}} , \quad (5)$$

where L_{QG} is the appropriate length scale that takes into account the correct values of magnitude and rate of the fluctuations.

If one wants to be open to the possibility that the nature of the stochastic processes associated to quantum space-time be not exactly (also see Section 8) the one of a random-walk model of quantum space-time fluctuations, then the f -dependence of the displacement amplitude spectral density could be different. This leads one to consider the more general parametrization

$$S(f) = f^{-\beta} c^{\beta-\frac{1}{2}} (\mathcal{L}_\beta)^{\frac{3}{2}-\beta} . \quad (6)$$

In this general parametrization the dimensionless quantity β carries the information on the nature of the underlying stochastic processes, while the length scale \mathcal{L}_β carries the information on the magnitude and rate of the fluctuations. I am assigning an index β to \mathcal{L}_β just in order to facilitate a concise description of experimental bounds; for example, if the fluctuations scenario with, say, $\beta = 0.6$ was ruled out down to values of the effective length scale of order, say, $10^{-27}m$ I would just write $\mathcal{L}_{\beta=0.6} < 10^{-27}m$. As I will discuss in Section 8, one might be interested in probing experimentally all values of β in the range $1/2 \leq \beta \leq 1$, with special interest in the cases $\beta = 1$ (the case of random-walk models whose effective length scale I denominated with $L_{QG} \equiv \mathcal{L}_{\beta=1}$), $\beta = 5/6$, and $\beta = 1/2$.

4.3 Comparison with gravity-wave interferometer data

Before discussing experimental bounds on \mathcal{L}_β from gravity-wave interferometers, let us fully appreciate the significance of these bounds by getting some intuition on the actual magnitude of the quantum fluctuations I am discussing. One intuition-building observation is that even for the case $\beta = 1$, which among the cases I consider is the one with the most virulent space-time fluctuations, the fluctuations predicted are truly minute: the $\beta = 1$ relation (2) only predicts fluctuations with standard deviation of order $10^{-5}m$ on a time of observation as large as 10^{10} years (the size of the whole observable universe is about 10^{10} light years!!). In spite of the smallness of these effects, the precision [47] of modern interferometers (the ones whose primary objective is the detection of the classical-gravity phenomenon of gravity waves) is such that we can obtain significant information at least on the scenarios with values of β toward the high end¹¹ of the interesting interval $1/2 \leq \beta \leq 1$, and in particular we can investigate quite sensitively the intuitive case of the random-walk model of space-time fluctuations. The operation of gravity-wave interferometers is based on the detection

¹¹As mentioned, for $L_{QG} = L_p$ the case $\beta = 1$ corresponds to a mean-square deviation induced by the distance fluctuations that is only linearly suppressed by L_p : $\sigma_D^2 \sim L_p c T$. Analogously, values of β in the interval $1/2 < \beta < 1$ correspond to σ_D^2 suppressed by a power of L_p between 1 and 2. The fact that we can only test values of β toward the high end of the interval $1/2 \leq \beta \leq 1$ can be intuitively characterized by stating that the fuzziness models we are able to test have σ_D^2 that is not much more than linearly suppressed by the Planck length.

of minute changes in the positions of some test masses (relative to the position of a beam splitter). If these positions were affected by quantum fluctuations of the type discussed above, the operation of gravity-wave interferometers would effectively involve an additional source of noise due to quantum gravity.

This observation allows to set interesting bounds already using existing noise-level data obtained at the *Caltech 40-meter interferometer*, which has achieved [50] displacement noise levels with amplitude spectral density lower than $10^{-18}m/\sqrt{Hz}$ for frequencies between 200 and 2000 Hz. While this is still very far from the levels required in order to probe significantly the lowest values of β (for $\mathcal{L}_{\beta=1/2} \sim L_p$ and $f \sim 1000Hz$ the quantum-gravity noise induced in the $\beta = 1/2$ scenario is only of order $10^{-36}m/\sqrt{Hz}$), these sensitivity levels clearly rule out all values of L_{QG} (*i.e.* $\mathcal{L}_{\beta=1}$) down to the Planck length. Actually, even values of L_{QG} significantly smaller than the Planck length are inconsistent with the data reported in Ref. [50]; in particular, from the observed noise level of $3 \cdot 10^{-19}m/\sqrt{Hz}$ near 450 Hz, which is the best achieved at the *Caltech 40-meter interferometer*, one obtains [7] the bound $L_{QG} \leq 10^{-40}m$. As discussed above, the random-walk model of distance fuzziness with fluctuations of magnitude L_p occurring at a rate of one per each t_p time interval, would correspond to the prediction $L_{QG} \sim L_p \sim 10^{-35}m$ and it is therefore **ruled out by these data**. This experimental information implies that, if one was to insist on this type of models, realistic random-walk models of quantum space-time fluctuations would have to be significantly less noisy (smaller prediction for L_{QG}) than the intuitive one which is now ruled out. Since, as I shall discuss, there are rather plausible scenarios for significantly less noisy random-walk models, it is important that experimentalists keep pushing forward the bound on L_{QG} . More stringent bounds on L_{QG} are within reach of the LIGO/VIRGO [48, 49] generation of gravity-wave interferometers.¹²

In planning future experiments, possibly tailored to test these effects (unlike LIGO and VIRGO which were tailored around the properties needed for gravity-wave detection), it is important to observe that an experiment achieving displacement noise levels with amplitude spectral density S^* at frequency f^* will set a bound on \mathcal{L}_β of order

$$\mathcal{L}_\beta < \left[S^* (f^*)^\beta c^{(1-2\beta)/2} \right]^{2/(3-2\beta)}, \quad (7)$$

which in particular for random-walk models takes the form

$$\mathcal{L}_\beta < \left[\frac{S^* f^*}{\sqrt{c}} \right]^2. \quad (8)$$

The structure of Eq. (7) (and Eq. (8)) shows that there can be instances in which a very large interferometer (the ideal tool for low-frequency studies) might not be better than a smaller interferometer, if the smaller one achieves a very small value of S^* .

The formula (7) can also be used to describe as a function of β the bounds on \mathcal{L}_β achieved by the data collected at the *Caltech 40-meter interferometer*. Using again the fact that a

¹²Besides allowing an improvement on the bound on L_{QG} intended as a universal property of quantum gravity, the LIGO/VIRGO generation of interferometers will also allow us to explore the idea that L_{QG} might be a scale that depends on the experimental context in such a way that larger interferometers pick up more of the space-time fluctuations. Based on the intuition coming from the Salecker-Wigner limit (here reviewed in Section 8), or just simply on phenomenological models in which distance fluctuations affect equally each L_p -long segment of a given distance, it would not be surprising if L_{QG} was a growing function of the length of the arms of the interferometer. This gives added significance to the step from the 40-meter arms of the existing Caltech interferometer to the few-Km arms of LIGO/VIRGO interferometers.

noise level of only $S^* \sim 3 \cdot 10^{-19} m / \sqrt{Hz}$ near $f^* \sim 450 Hz$ was achieved [50], one obtains the bounds

$$[\mathcal{L}_\beta]_{caltech} < \left[\frac{3 \cdot 10^{-19} m}{\sqrt{Hz}} (450 Hz)^\beta c^{(1-2\beta)/2} \right]^{2/(3-2\beta)}. \quad (9)$$

Let me comment in particular on the case $\beta = 5/6$ which might deserve special attention because of its connection (which was derived in Refs. [7, 24] and will be reviewed here in Section 8) with certain arguments for bounds on the measurability of distances in quantum gravity [24, 45, 43]. From Eq. (9) we find that $\mathcal{L}_{\beta=5/6}$ is presently bound to the level $\mathcal{L}_{\beta=5/6} \leq 10^{-29} m$. This bound is remarkably stringent in absolute terms, but is still quite far from the range of values one ordinarily considers as likely candidates for length scales appearing in quantum gravity. A more significant bound on $\mathcal{L}_{\beta=5/6}$ should be obtained by the LIGO/VIRGO generation of gravity-wave interferometers. For example, it is plausible [48] that the “advanced phase” of LIGO achieve a displacement noise spectrum of less than $10^{-20} m / \sqrt{Hz}$ near 100 Hz and this would probe values of $\mathcal{L}_{\beta=5/6}$ as small as $10^{-34} m$.

Interferometers are our best long-term hope for the development of this phenomenology, and that is why the analysis in this Section focuses on interferometers. However, it should be noted that among detectors already in operation the best bound on L_β (if taken as universal) comes from resonant-bar detectors such as NAUTILUS [52], which achieved displacement sensitivity of about $10^{-21} m / \sqrt{Hz}$ near 924 Hz. Correspondingly, one obtains the bound

$$[\mathcal{L}_\beta]_{bars} < \left[\frac{10^{-21} m}{\sqrt{Hz}} (924 Hz)^\beta c^{(1-2\beta)/2} \right]^{2/(3-2\beta)}. \quad (10)$$

In closing this subsection on interferometry data analysis relevant for space-time fuzziness scenarios, let me clarify how it happened that such small effects could be tested. As I already mentioned, one of the viable strategies for quantum-gravity experiments is the one finding ways to put together very many of the very small quantum-gravity effects. In these interferometric studies that I proposed in Ref. [7] one does indeed effectively sum up a large number of quantum space-time fluctuations. In a time of observation as long as the inverse of the typical gravity-wave interferometer frequency of operation an extremely large number of minute quantum fluctuations could affect the distance between the test masses. Although these fluctuations average out, they do leave traces in the interferometer. These traces grow with the time of observation: the standard deviation increases with the time of observation, while the displacement noise amplitude spectral density increases with the inverse of frequency (which again effectively means that it increases with the time of observation). From this point of view it is not surprising that plausible quantum-gravity scenarios ($1/2 \leq \beta \leq 1$) all involve higher noise at lower frequencies: the observation of lower frequencies requires longer times and is therefore affected by a larger number of quantum-gravity fluctuations.

4.4 Less noisy random-walk models of distance fluctuations?

The most intuitive result obtained in Refs. [7, 24] and reviewed in the preceding subsection is that we can rule out the picture in which the distances between the test masses of the interferometer are affected by fluctuations of magnitude L_p occurring at a rate of one per each t_p time interval. Does this rule out completely the possibility of a random-walk model of distance fluctuations? or are we just learning that the most intuitive/naive example of such a model does not work, but there are other plausible random-walk models?

Without wanting to embark on a discussion of the plausibility of less noisy random-walk models, I shall nonetheless discuss some ideas which could lead to this noise reduction.

Let me start by observing that certain studies of measurability of distances in quantum gravity (see Ref. [24] and the brief review of those arguments which is provided in parts of Section 8) can be interpreted as suggesting that L_{QG} might not be a universal length scale, *i.e.* it might depend on some specific properties of the experimental setup (particularly the energies/masses involved), and in some cases L_{QG} could be significantly smaller than L_p .

Another possibility one might want to consider [24] is the one in which the quantum properties of space-time are such that fluctuations of magnitude L_p would occur with frequency somewhat lower than $1/t_p$. This might happen for various reasons, but a particularly intriguing possibility¹³ is the one of theories whose fundamental objects are not pointlike, such as the popular string theories. For such theories it is plausible that fluctuations occurring at the Planck-distance level might have only a modest impact on extended fundamental objects characterized by a length scale significantly larger than the Planck length (*e.g.* in string theory the string size, or “length”, might be an order of magnitude larger than the Planck length). This possibility is interesting in general for quantum-gravity theories with a hierarchy of length scales, such as certain “M-theory motivated” scenarios with an extra length scale associated to the compactification from 11 to 10 dimensions.

Yet another possibility for a random-walk model to cause less noise in interferometers could emerge if somehow the results of the schematic analysis adopted here and in Refs. [7, 24] turned out to be significantly modified once we become capable of handling all of the details of a real interferometer. To clarify which type of details I have in mind let me mention as an example the fact that in my analysis the structure of the test masses was not taken into account in any way: they were essentially treated as point-like. It would not be too surprising if we eventually became able to construct theoretical models taking into account the interplay between the binding forces that keep together (“in one piece”) a macroscopic test mass as well as some random-walk-type fundamental fluctuations of the space-time in which these macroscopic bodies “live”. The interference pattern observed in the laboratory reflects the space-time fluctuations only filtered through their interplay with the mentioned binding forces of the macroscopic test masses. These open issues are certainly important and a lot of insight could be gained through their investigation, but there is also some confusion that might easily result¹⁴ from simple-minded considerations (possibly guided by intuition developed using rudimentary table-top interferometers) concerning the macroscopic nature of the test masses used in modern interferometers. In closing this section let me try to offer a few relevant clarifications. I need to start by adding some comments on the stochastic processes I have been considering. In most physical contexts a series of random steps does not lead to $\sqrt{T_{obs}}$ dependence of σ because often the context is such that through the fluctuation-dissipation theorem the source of $\sqrt{T_{obs}}$ dependence is (partly) compensated (some sort of restoring effect). The hypothesis explored in these discussions of random-walk models of space-time fuzziness is that the type of underlying dynamics of quantum space-time be such that the fluctuation-dissipation theorem be satisfied without spoiling the $\sqrt{T_{obs}}$ dependence of σ . This is an intuition which apparently is shared by other authors; for example, the study reported in Ref. [55] (which followed by a few months Ref. [7], but clearly was the result of completely independent work) also models some implications of quantum space-time (the ones that affect clocks) with stochastic processes whose underlying dynamics does not

¹³This possibility emerged in discussions with Gabriele Veneziano. In response to my comments on the possibility of fluctuations with frequency somewhat lower than $1/t_p$ Gabriele made the suggestion that extended fundamental objects might be less susceptible than point particles to very localized space-time fluctuations. It would be interesting to work out in some detail an example of dynamical model of strings in a fuzzy space-time.

¹⁴In particular, as I emphasized in Ref. [53], these and other elements of confusion are responsible for the incorrect conclusions on the Salecker-Wigner measurability limit which were drawn in the very recent Ref. [54].

produce any dissipation and therefore the “fluctuation contribution” to the T_{obs} dependence is left unmodified, although the fluctuation-dissipation theorem is fully taken into account.

Since a mirror of an interferometer of LIGO/VIRGO type is in practice an extremity of a pendulum, another aspect that the reader might at first find counter-intuitive is the fact that the $\sqrt{T_{obs}}$ dependence of σ , although coming in with a very small prefactor, for extremely large T_{obs} would seem to give values of σ too large to be consistent with the structure of a pendulum. This is a misleading intuition which originates from the experience with ordinary (non-quantum-gravity) analyses of the pendulum. In fact, the dynamics of an ordinary pendulum has one extremity “fixed” to a very heavy macroscopic and rigid body, while the other extremity is fixed to a much lighter (but, of course, still macroscopic) body. The usual stochastic processes considered in the study of the pendulum affect the heavier body in a totally negligible way, while they have strong impact on the dynamics of the lighter body. A pendulum analyzed according to a random-walk model of space-time fluctuations would be affected by stochastic processes which are of the same magnitude both for its heavier and its lighter extremity. [The bodies are fluctuating along with the intrinsic space-time fluctuations, rather than fluctuating as a result of, say, collisions with material particles occurring in a conventional space-time.] In particular, in the directions orthogonal to the vertical axis the stochastic processes affect the position of the center of mass of the entire pendulum just as they would affect the position of the center of mass of any other body (the spring that connects the two extremities of the pendulum would not affect the motion of the overall center of mass of the pendulum). With respect to the application of some of these considerations to modern gravity-wave interferometers it is also important to keep in mind that the measurement strategy of these interferometers requires that their test masses be free-falling.

5 GAMMA-RAY BURSTS AND IN-VACUO DISPERSION

Let me now discuss the proposal put forward in Ref. [5] (also see Ref. [56]), which exploits the recent confirmation that at least some gamma-ray bursters are indeed at cosmological distances [57, 58, 59, 60], making it possible for observations of these to provide interesting constraints on the fundamental laws of physics. In particular, such cosmological distances combine with the short time structure seen in emissions from some GRBs [61] to provide ideal features for tests of possible *in vacuo* dispersion of electromagnetic radiation from GRBs, of the type one might expect based on the intuitive quantum-gravity arguments reviewed in Section 2. As mentioned, a quantum-gravity-induced deformation of the dispersion relation for photons would naturally take the form $c^2 \mathbf{p}^2 = E^2 [1 + \mathcal{F}(E/E_{QG})]$, where E_{QG} is an effective quantum-gravity energy scale and \mathcal{F} is a model-dependent function of the dimensionless ratio E/E_{QG} . In quantum-gravity scenarios in which the Hamiltonian equation of motion $\dot{x}_i = \partial H / \partial p_i$ is still valid (at least approximately valid; valid to an extent sufficient to justify the analysis that follows) such a deformed dispersion relation would lead to energy-dependent velocities for massless particles, with implications for the electromagnetic signals that we receive from astrophysical objects at large distances. At small energies $E \ll E_{QG}$, it is reasonable to expect that a series expansion of the dispersion relation should be applicable leading to the formula (1). For the case $\alpha = 1$, which is the most optimistic (largest quantum-gravity effect) among the cases discussed in the quantum-gravity literature, the formula (1) reduces to

$$c^2 \mathbf{p}^2 \simeq E^2 \left(1 + \xi \frac{E}{E_{QG}} \right) . \quad (11)$$

Correspondingly one would predict the energy-dependent velocity formula

$$v = \frac{\partial E}{\partial p} \sim c \left(1 - \xi \frac{E}{E_{QG}} \right) . \quad (12)$$

To elaborate a bit more than I did in Section 2 on the intuition that leads to this type of candidate quantum-gravity effect let me observe that [5] velocity dispersion such as described in (12) could result from a picture of the vacuum as a quantum-gravitational ‘medium’, which responds differently to the propagation of particles of different energies and hence velocities. This is analogous to propagation through a conventional medium, such as an electromagnetic plasma [62]. The gravitational ‘medium’ is generally believed to contain microscopic quantum fluctuations, such as the ones considered in the previous sections. These may [63] be somewhat analogous to the thermal fluctuations in a plasma, that occur on time scales of order $t \sim 1/T$, where T is the temperature. Since it is a much ‘harder’ phenomenon associated with new physics at an energy scale far beyond typical photon energies, any analogous quantum-gravity effect could be distinguished by its different energy dependence: the quantum-gravity effect would increase with energy, whereas conventional medium effects decrease with energy in the range of interest [62].

Also relevant for building some quantum-gravity intuition for this type of *in vacuo* dispersion and deformed velocity law is the observation [46, 23] that this has implications for the measurability of distances in quantum gravity that fit well with the intuition emerging from heuristic analyses [12] based on a combination of arguments from ordinary quantum mechanics and general relativity. [This connection between dispersion relations and measurability bounds will be here reviewed in Section 8.]

Notably, recent work [41] has provided evidence for the possibility that the popular canonical/loop quantum gravity [40] might be among the theoretical approaches that admit the phenomenon of deformed dispersion relations with the deformation going linearly with the Planck length ($L_p \sim 1/E_p$). Similarly, evidence for this type of dispersion relations has been found [46] in *Liouville* (non-critical) strings [19], whose development was partly motivated by an intuition concerning the “quantum-gravity vacuum” that is rather close to the one traditionally associated to the works of Wheeler [14] and Hawking [15]. Moreover, the phenomenon of deformed dispersion relations with the deformation going linearly with the Planck length fits rather naturally within certain approaches based on non-commutative geometry and deformed symmetries. In particular, there is growing evidence [23, 27, 28] for this phenomenon in theories living in the non-commutative Minkowski space-time proposed in Refs. [64, 65, 21], which involves a dimensionful (presumably Planck-length related) deformation parameter.

Equation (12) encodes a minute modification for most practical purposes, since E_{QG} is believed to be a very high scale, presumably of order the Planck scale E_p . Nevertheless, such a deformation could be rather significant for even moderate-energy signals, if they travel over very long distances. According to (12) two signals respectively of energy E and $E + \Delta E$ emitted simultaneously from the same astrophysical source in traveling a distance L acquire a “relative time delay” $|\delta t|$ given by

$$|\delta t| \sim \frac{\Delta E}{E_{QG}} \frac{L}{c} . \quad (13)$$

Such a time delay can be observable if ΔE and L are large while the time scale over which the signal exhibits time structure is small. As mentioned, these are the respects in which GRBs offer particularly good prospects for such measurements. Typical photon energies in GRB emissions are in the range $0.1 - 100$ MeV [61], and it is possible that the spectrum might in fact extend up to TeV energies [66]. Moreover, time structure down to the millisecond scale has been observed in the light curves [61], as is predicted in the most popular theoretical

models [67] involving merging neutron stars or black holes, where the last stages occur on the time scales associated with grazing orbits. Similar time scales could also occur in models that identify GRBs with other cataclysmic stellar events such as failed supernovae Ib, young ultra-magnetized pulsars or the sudden deaths of massive stars [68]. We see from equations (12) and (13) that a signal with millisecond time structure in photons of energy around 10 MeV coming from a distance of order 10^{10} light years, which is well within the range of GRB observations and models, would be sensitive to E_{QG} of order the Planck scale.

In order to set a definite bound on E_{QG} it is necessary to measure L and to measure the time of arrival of different energy/wavelength components of a sharp peak within the burst. From Eq. (13) it follows that one could set a bound

$$E_{QG} > \Delta E \frac{L}{c|\tau|} \quad (14)$$

by establishing the times of arrival of the peak to be the same up to an uncertainty τ in two energy channels E and $E + \Delta E$. Unfortunately, at present we have data available only on a few GRBs for which the distance L has been determined (the first measurements of this type were obtained only in 1997), and these are the only GRBs which can be reliably used to set bounds on the new effect. Moreover, mostly because of the nature of the relevant experiments (particularly the BATSE detector on the Compton Gamma Ray Observatory), for a large majority of the GRBs on record only the portion of the burst with energies up to the MeV energy scale was observed, whereas higher energies would be helpful for the study of the phenomenon of quantum-gravity-induced dispersion here considered (which increases linearly with energy). We expect significant improvements in these coming years. The number of observed GRBs with associated distance measurement should rapidly increase. A new generation of orbiting spectrometers, *e.g.* AMS [69] and GLAST [70], are being developed, whose potential sensitivities are very impressive. For example, assuming a E^{-2} energy spectrum, GLAST would expect to observe about 25 GRBs per year at photon energies exceeding 100 GeV, with time resolution of microseconds. AMS would observe a similar number at $E > 10$ GeV with time resolution below 100 nanoseconds.

While we wait for these new experiments, preliminary bounds can already be set with available data. Some of these bounds are “conditional” in the sense that they rely on the assumption that the relevant GRB originated at distances corresponding to redshift of $\mathcal{O}(1)$ (corresponding to a distance of ~ 3000 Mpc), which appears to be typical. Let me start by considering the “conditional” bound (first considered in Ref. [5]) which can be obtained from data on GRB920229. GRB920229 exhibited [71] micro-structure in its burst at energies up to ~ 200 KeV. In Ref. [5] it was estimated conservatively that a detailed time-series analysis might reveal coincidences in different BATSE energy bands on a time-scale $\sim 10^{-2}$ s, which, assuming redshift of $\mathcal{O}(1)$ (the redshift of GRB920229 was not measured) would yield sensitivity to $E_{QG} \sim 10^{16}$ GeV (it would allow to set a bound $E_{QG} > 10^{16}$ GeV).

As observed in Ref. [56], a similar sensitivity might be obtainable with GRB980425, given its likely identification with the unusual supernova 1998bw [72]. This is known to have taken place at a redshift $z = 0.0083$ corresponding to a distance $D \sim 40$ Mpc (for a Hubble constant of $65 \text{ km sec}^{-1} \text{ Mpc}^{-1}$) which is rather smaller than a typical GRB distance. However GRB980425 was observed by BeppoSAX at energies up to 1.8 MeV, which gains back an order of magnitude in the sensitivity. If a time-series analysis were to reveal structure at the $\Delta t \sim 10^{-3}$ s level, which is typical of many GRBs [73], it would yield the same sensitivity as GRB920229 (but in this case, in which a redshift measurement is available, one would have a definite bound, whereas GRB920229 only provides a “conditional” bound of the type discussed above).

Ref. [56] also observed that an interesting (although not very “robust”) bound could be obtained using GRB920925c, which was observed by WATCH [74] and possibly in high-energy γ rays by the HEGRA/AIROBICC array above 20 TeV [75]. Several caveats are in

order: taking into account the appropriate trial factor, the confidence level for the signal seen by HEGRA to be related to GRB920925c is only 99.7% ($\sim 2.7\sigma$), the reported directions differ by 9° , and the redshift of the source is unknown. Nevertheless, the potential sensitivity is impressive. The events reported by HEGRA range up to $E \sim 200$ TeV, and the correlation with GRB920925c is within $\Delta t \sim 200$ s. Making the reasonably conservative assumption that GRB920925c occurred no closer than GRB980425, viz. ~ 40 Mpc, one finds a minimum sensitivity to $E_{QG} \sim 10^{19}$ GeV, modulo the caveats listed above. Even more spectacularly, several of the HEGRA GRB920925c candidate events occurred within $\Delta t \sim 1$ s, providing a potential sensitivity even two orders of magnitude higher.

As illustrated by this discussion, the GRBs have remarkable potential for the study of *in vacuo* dispersion, which will certainly lead to impressive bounds/tests as soon as improved experiments are put into operation, but at present the best GRB-based bounds are either “conditional” (example of GRB92022) or “not very robust” (example of GRB920925c). As a result, at present the best (reliable) bound has been extracted [26, 76] using data from the Whipple telescope on a TeV γ -ray flare associated with the active galaxy Mrk 421. This object has a redshift of 0.03 corresponding to a distance of ~ 100 Mpc. Four events with γ -ray energies above 2 TeV have been observed within a period of 280 s. These provide [26, 76] a definite limit $E_{QG} > 4 \times 10^{16}$ GeV.

In passing let me mention that (as observed in Ref. [5, 46]) pulsars and supernovae, which are among the other astrophysical phenomena that might at first sight appear well suited for the study of *in vacuo* dispersion, do not actually provide interesting sensitivities. Although pulsar signals have very well-defined time structure, they are at relatively low energies and are presently observable over distances of at most 10^4 light years. If one takes an energy of order 1 eV and postulates generously a sensitivity to time delays as small as $1 \mu\text{sec}$, one nevertheless reaches only an estimated sensitivity to $E_{QG} \sim 10^9$ GeV. With new experiments such as AXAF it may be possible to detect X-ray pulsars out to 10^6 light years, but this would at best push the sensitivity up to $E_{QG} \sim 10^{11}$ GeV. Concerning supernovae, it is important to take into account that neutrinos¹⁵ from Type II events similar to SN1987a, which should have energies up to about 100 MeV with a time structure that could extend down to milliseconds, are likely to be detectable at distances of up to about 10^5 light years, providing sensitivity to $E_{QG} \sim 10^{15}$ GeV, which is remarkable in absolute terms, but is still significantly far from the Planck scale and anyway cannot compete with the type of sensitivity achievable with GRBs.

It is rather amusing that GRBs can provide such a good laboratory for investigations of *in vacuo* dispersion in spite of the fact that the short-time structure of GRB signals is still not understood. The key point of the proposal in Ref. [5] is that sensitive tests can be performed through the serendipitous detection of short-scale time structure [71] at different energies in GRBs which are established to be at cosmological distances. Detailed features of burst time series enable (as already shown in several examples) the emission times in different energy ranges to be put into correspondence. Any time shift due to quantum-gravity would *increase* with the photon energy, and this characteristic dependence is separable from more conventional in-medium-physics effects, which *decrease* with energy. To distinguish any quantum-gravity induced shift from effects due to the source, one can use the fact that the quantum-gravity effect here considered is *linear* in the GRB distance.

This last remark applies to all values of α , but most of the observations and formulas in this section are only valid in the case $\alpha = 1$ (linear suppression). The generalization to cases

¹⁵Of course, at present we should be open to the possibility that the velocity law (12) might apply to all massless particles, but it is also plausible that the correct quantum-gravity velocity law would depend on the spin of the particle. It would therefore be important to set up a phenomenological programme that studies neutrinos with the same level of sensitivities that GRBs and other astrophysical phenomena allow for the study of the velocity law of the photon.

with $\alpha \neq 1$ is however rather simple; for example, Eq. (14) takes the form (up to coefficients of order 1)

$$E_{QG} > \left[[(E + \Delta E)^\alpha - E^\alpha] \frac{L}{c|\tau|} \right]^{1/\alpha}. \quad (15)$$

Notice that here, because of the non-linearity, the right-hand side depends both on E and ΔE .

Before moving on to other experiments let me clarify what is the key ingredient of this experiment using observations of gamma rays from distant astrophysical sources (the ingredient that allowed to render observable the minute quantum-gravity effects). This ingredient is very similar to the one relevant for the studies of space-time fuzziness using modern interferometers which I discussed in the preceding section; in fact, the gamma rays here considered are affected by a very large number of the minute quantum-gravity effects. Each of the dispersion-inducing quantum-gravity effect is very small, but the gamma rays emitted by distant astrophysical sources travel for a very long time before reaching us and can therefore be affected by an extremely large number of such effects.

6 OTHER QUANTUM-GRAVITY EXPERIMENTS

In this section I provide brief reviews of some other quantum-gravity experiments. The fact that the discussion here provided for these experiments is less detailed than the preceding discussions of the interferometry-based and GRB-based experiments is not to be interpreted as indicating that these experiments are somehow less significant: it is just that a detailed discussion of a couple of examples was sufficient to provide to the reader some general intuition on the strategy behind quantum-gravity experiments and it was natural for me to use as examples the ones I am most familiar with. For the experiments discussed in this section I shall just give a rough idea of the quantum-gravity scenarios that could be tested and of the experimental procedures which have been proposed.

6.1 Neutral kaons and CPT violation

One of the formalisms that has been proposed [17, 2] for the evolution equations of particles in the space-time foam relies on a density-matrix picture. The foam is seen as providing a sort of environment inducing quantum decoherence even on isolated systems (*i.e.* systems which only interact with the foam). A given non-relativistic system (such as the neutral kaons studied by the CPLEAR collaboration at CERN) is described by a density matrix ρ that satisfies an evolution equation analogous to the one ordinarily used for the quantum mechanics of certain open systems:

$$\partial_t \rho = i[\rho, H] + \delta H \rho \quad (16)$$

where H is the ordinary Hamiltonian and δH , which has a non-commutator structure [2], represents the effects of the foam. δH is expected to be extremely small, suppressed by some power of the Planck length. The precise form of δH (which in particular would set the level of the new physics by establishing how many powers of the Planck length suppress the effect) has not yet been derived from some full-grown quantum gravity¹⁶, and therefore phenomenological parametrizations have been introduced (see Refs. [17, 77, 20, 35]). For

¹⁶Within the quantum-gravity approach here reviewed in Subsection 11.2, which only attempts to model certain aspects of quantum gravity, such a direct calculation might soon be performed.

the case in which the effects are only suppressed by one power of the Planck length (linear suppression) recent neutral-kaon experiments, such as the ones performed by CPLEAR, have set significant bounds [2] on the associated CPT-violation effects and forthcoming experiments are likely to push these bounds even further.

Like the interferometry-based and the GRB-based experiments, these experiments (which have the added merit of having started the recent wave of quantum-gravity proposals) also appear to provide significant quantum-gravity tests. As mentioned, the effect of quantum-gravity-induced decoherence certainly qualifies as a traditional quantum-gravity subject, and the level of sensitivity reached by the neutral-kaon studies is certainly significant (as in the case of *in vacuo* dispersion and GRBs, one would like to be able to explore also the case of a quadratic Planck-length suppression, but it is nonetheless very significant that we have at least reached the capability to test the case of linear suppression). Also in this case it is natural to ask: how come we could manage this? What strategy allowed this neutral-kaon studies to evade the traditional gloomy forecasts for quantum-gravity phenomenology? While, as discussed above, in the interferometry-based and the GRB-based experiments the crucial element in the experimental proposal is the possibility to put together many quantum gravity effects, in the case of the neutral-kaon experiments the crucial element in the experimental proposal is provided by the very delicate balance of scales that characterizes the neutral-kaon system. In particular, it just happens to be true that the dimensionless ratio setting the order of magnitude of quantum-gravity effects in the linear suppression scenario, which is $c^2 M_{L,S}/E_p \sim 2 \cdot 10^{-19}$, is not much smaller than some of the dimensionless ratios characterizing the neutral-kaon system, notably the ratio $|M_L - M_S|/M_{L,S} \sim 7 \cdot 10^{-15}$ and the ratio $|\Gamma_L - \Gamma_S|/M_{L,S} \sim 1.4 \cdot 10^{-14}$. This renders possible for the quantum-gravity effects to provide observably large corrections to the physics of neutral kaons.

6.2 Interferometry and string cosmology

Up to this point I have only reviewed experiments probing foamy properties of space-time in the sense of Wheeler and Hawking. A different type of quantum-gravity effect which might produce a signature strong enough for experimental testing has been discussed in the context of studies of a cosmology based on critical superstrings [78]. While for a description of this cosmology and of its physical signatures I must only refer the reader to the recent reviews in Ref. [79], I want to briefly discuss here the basic ingredients of the proposal [3] of interferometry-based tests of the stochastic gravity-wave background predicted by string cosmology.

In string cosmology the universe starts from a state of very small curvature, then goes through a long phase of dilaton-driven inflation reaching nearly Planckian energy density, and then eventually reaches the standard radiation-dominated cosmological evolution [78, 79]. The period of nearly Planckian energy density plays a crucial role in rendering the quantum-gravity effects observable. In fact, this example based on string cosmology is quite different from the experiments I discussed earlier in these lectures also because it does not involve small quantum-gravity effects which are somehow amplified (in the sense for example of the amplification provided when many effects are somehow put together). The string cosmology involves a period in which the quantum-gravity effects are actually quite large. As clarified in Refs. [78, 79] planned gravity-wave detectors such as LIGO might be able to detect the faint residual traces of these strong effects occurred in a far past.

As mentioned, the quantum-gravity effects that, within string cosmology, leave a trace in the gravity-wave background are not of the type that requires an active Wheeler-Hawking foam. The relevant quantum-gravity effects live in the more familiar vacuum which we are used to encounter in the context of ordinary gauge theory. (Actually, for the purposes of the analyses reported in Refs. [78, 79] quantum gravity could be seen as an ordinary gauge theory, although with unusual gauge-field content.) In the case of the Wheeler-Hawking foam one is tempted to visualize the vacuum as reboiling with (virtual) worm-holes and

black-holes. Instead the effects relevant for the gravity-wave background predicted by string cosmology are more conventional field-theory-type fluctuations, although carrying gravitational degrees of freedom, like the graviton. Also from this point of view the experimental proposal discussed in Refs. [78, 79] probes a set of candidate quantum-gravity phenomena which is complementary to the ones I have reviewed earlier in these notes.

6.3 Matter interferometry and primary state diffusion

The studies reported in Ref. [4] (and references therein) have considered how certain effectively stochastic properties of space-time would affect the evolution of quantum-mechanical states. The stochastic properties there considered are different from the ones discussed here in Sections 2, 3 and 4, but were introduced within a similar viewpoint, *i.e.* stochastic processes as effective description of quantum space-time processes. The implications of these stochastic properties for the evolution of quantum-mechanical states were modeled via the formalism of “primary state diffusion”, but only rather crude models turned out to be treatable.

The approach proposed in Ref. [4] actually puts together some of the unknowns of space-time foam and the specific properties of “primary state diffusion”. The structure of the predicted effects cannot be simply discussed in terms of elementary properties of space-time foam and a simple interpretation in terms of symmetry deformations does not appear to be possible. Those effects appear to be the net result of the whole formalism that goes into the approach. Moreover, as also emphasized by the authors, the crudeness of the models is such that all conclusions are to be considered as tentative at best. Still, the analysis reported in Ref. [4] is very significant as an independent indication of a mechanism, based on matter-interferometry experiments, that could unveil Planck-length-suppressed effects.

6.4 Colliders and large extra dimensions

It was recently suggested [80, 81] that the characteristic quantum-gravity length scale might be given by a length scale L_D much larger than the Planck length in theories with large extra dimensions. It appears plausible that there exist models that are consistent with presently-available experimental data and have L_D as large as the $(TeV)^{-1}$ scale and (some of) the extra dimensions as large as a millimeter [81]. In such models the smallness of the Planck length is seen as the result of the fact that the strength of gravitation in the ordinary 3+1 space-time dimensions would be proportional to the square-root of the inverse of the large volume of the external compactified space multiplied by an appropriate (according to dimensional analysis) power of L_D .

Several studies have been motivated by the proposal put forward in Ref. [81], but only a small percentage of these studies considered the implications for quantum-gravity scenarios. Among these studies the ones reported in Refs. [8, 9] are particularly significant for the objectives of these lectures, since they illustrate another completely different strategy for quantum-gravity experiments. It is there observed that within the realm of the ordinary 3+1 dimensional space-time an important consequence of the existence of large extra dimensions would be the presence of a tower of Kaluza-Klein modes associated to the gravitons. The weakness of the coupling between gravitons and other particles can be compensated by the large number of these Kaluza-Klein modes when the experimental energy resolution is much larger than the mass splitting between the modes, which for a small number of very large extra dimensions can be a weak requirement (*e.g.* for 6 millimeter-wide extra dimensions [81, 8] the mass splitting is of a few MeV). This can lead to observably large [8, 9] effects at planned particle-physics colliders, particularly CERN’s LHC.

In a sense, the experimental proposal put forward in Refs. [8, 9] is another example of experiment in which the smallness of quantum gravity effects is compensated by putting

together a large number of such effects (putting together the contributions of all of the Kaluza-Klein modes).

Concerning the quantum-gravity aspects of the models with large extra dimensions proposed in Ref. [81], it is important to realize that, as emphasized in Ref. [24], if anything like the space-time foam here described in Sections 2, 3, 4 and 5 was present in such models the effective reduction of the quantum-gravity scale would naturally lead to foamy effects that are too large for consistency with available experimental data. Preliminary estimates based solely on dimensional considerations appear to suggest that [24] linear suppression by the reduced quantum-gravity scale would certainly be ruled out and even quadratic suppression might not be sufficient for consistency with available data. These arguments should lead to rather stringent bounds on space-time foam especially in those models in which some of the large extra dimensions are accessible to non-gravitational particles (see, *e.g.*, Ref. [82]), and should have interesting (although smaller) implications also for the popular scenario in which only the gravitational degrees of freedom have access to the large extra dimensions. Of course, a final verdict must await detailed calculations analysing space-time foam in these models with large extra dimensions. The first examples of this type of computations are given by the very recent studies in Refs. [83, 84], which considered the implications of foam-induced light-cone deformation for certain examples of models with large extra dimensions.

7 CLASSICAL-SPACE-TIME-INDUCED QUANTUM PHASES IN MATTER INTERFEROMETRY

While of course the quantum properties of space-time are the most exciting effects we expect of quantum gravity, and probably the ones which will prove most useful in gaining insight into the fundamental structure of the theory, it is important to investigate experimentally all aspects of the interplay between gravitation and quantum mechanics. Among these experiments the ones that could be expected to provide fewer surprises (and less insight into the structure of quantum gravity) are the ones concerning the interplay between strong-but-classical gravitational fields and quantum matter fields. However, this is not necessarily true as I shall try to clarify within this section's brief review of the experiment performed nearly a quarter of a century ago by Colella, Overhauser and Werner [10], which, to my knowledge, was the first experiment probing some aspect of the interplay between gravitation and quantum mechanics. That experiment has been followed by several modifications and refinements (often labeled "COW experiments" from the initials of the scientists involved in the first experiment) all probing the same basic physics, *i.e.* the validity of the Schrödinger equation

$$\left[- \left(\frac{\hbar^2}{2 M_I} \right) \vec{\nabla}^2 + M_G \phi(\vec{r}) \right] \psi(t, \vec{r}) = i \hbar \frac{\partial \psi(t, \vec{r})}{\partial t} \quad (17)$$

for the description of the dynamics of matter (with wave function $\psi(t, \vec{r})$) in presence of the Earth's gravitational potential $\phi(\vec{r})$. [In (17) M_I and M_G denote the inertial and gravitational mass respectively.]

The COW experiments exploit the fact that the Earth's gravitational potential puts together the contribution of so many particles (all of those composing the Earth) that it ends up being large enough to introduce observable effects in rotating table-top interferometers. This was the first example of a physical context in which gravitation was shown to have an observable effect on a quantum-mechanical system in spite of the weakness of the gravitational force.

The fact that the original experiment performed by Colella, Overhauser and Werner obtained results in very good agreement [10] with Eq. (17) might seem to indicate that, as

generally expected, experiments on the interplay between strong-but-classical gravitational fields and quantum matter fields should not lead to surprises and should not provide insight into the structure of quantum gravity. However, the confirmation of Eq. (17) does raise some sort of a puzzle with respect to the Equivalence Principle of general relativity; in fact, even for $M_I = M_G$ the mass does not cancel out in the quantum evolution equation (17). This is an observation that by now has also been emphasized in textbooks [85], but to my knowledge it has not been fully addressed even within the most popular quantum-gravity approaches, *i.e.* critical superstrings and canonical/loop quantum gravity. Which role should be played by the Equivalence Principle in quantum gravity? Which version/formulation of the Equivalence Principle should/could hold in quantum gravity?

Additional elements for consideration in quantum-gravity models will emerge if the small discrepancy between (17) and data reported in Ref. [86] (a refined COW experiment) is confirmed by future experiments. The subject of gravitationally induced quantum phases is also expanding in new directions [6, 87], which are likely to provide additional insight.

8 ESTIMATES OF SPACE-TIME FUZZINESS FROM MEASURABILITY BOUNDS

In the preceding Sections 4, 5, 6 and 7 I have discussed the experimental proposals that support the conclusions anticipated in Sections 2 and 3. This Section 8 and the following three sections each provide a “theoretical-physics addendum”. In this section I discuss some arguments that appear to suggest properties of the space-time foam. These arguments are based on analyses of bounds on the measurability of distances in quantum gravity. The existence of measurability bounds has attracted the interest of several theorists, because these bounds appear to capture an important novel element of quantum gravity. In ordinary (non-gravitational) quantum mechanics there is no absolute limit on the accuracy of the measurement of a distance. [Ordinary quantum mechanics allows $\delta A = 0$ for any single observable A , since it only limits the combined measurability of pairs of conjugate observables.]

The quantum-gravity bound on the measurability of distances (whatever final form it actually takes in the correct theory) is of course intrinsically interesting, but here (as in previous works [7, 24, 12, 88, 13]) I shall be interested in the possibility that it might reflect properties of the space-time foam. This is of course not necessarily true: a bound on the measurability of distances is not necessarily associated to space-time fluctuations, but guided by the Wheeler-Hawking intuition on the nature of space-time one is tempted to interpret any measurability bound (which might be obtained with totally independent arguments) as an indicator of the type of irreducible fuzziness that characterizes space-time. One has on one hand some intuition about quantum gravity which involves stochastic fluctuations of distances and on the other hand some different arguments lead to intuition for absolute bounds on the measurability of distances; it is natural to explore the possibility that the two might be related, *i.e.* that the intrinsic stochastic fluctuations should limit the measurability just to the level suggested by the independent measurability arguments. Different arguments appear to lead to different measurability bounds, which in turn could provide different intuition for the stochastic properties of space-time foam.

8.1 Minimum-length noise

In many quantum-gravity approaches there appears to be a length scale L_{min} , often identified with the Planck length or the string length L_{string} (which, as mentioned, should be somewhat larger than the Planck length, plausibly in the neighborhood of $10^{-34}m$), which sets an

absolute bound on the measurability of distances (a minimum uncertainty):

$$\delta D \geq L_{min} . \quad (18)$$

This property emerges in approaches based on canonical quantization of Einstein's gravity when analyzing certain gedanken experiments (see, *e.g.*, Refs. [30, 33] and references therein). In critical superstring theories, theories whose mechanics is still governed by the laws of ordinary quantum mechanics but with one-dimensional (rather than point-like) fundamental objects, a relation of type (18) follows from the stringy modification of Heisenberg's uncertainty principle [31]

$$\delta x \delta p \geq 1 + L_{string}^2 \delta p^2 . \quad (19)$$

In fact, whereas Heisenberg's uncertainty principle allows $\delta x = 0$ (for $\delta p \rightarrow \infty$), for all choices of δp the uncertainty relation (19) gives $\delta x \geq L_{string}$. The relation (19) is suggested by certain analyses of string scattering [31], but it might have to be modified when taking into account the non-perturbative solitonic structures of superstrings known as Dirichlet branes [38]. In particular, evidence has been found [89] in support of the possibility that "Dirichlet particles" (Dirichlet 0 branes) could probe the structure of space-time down to scales shorter than the string length. In any case, all evidence available on critical superstrings is consistent with a relation of type (18), although it is probably safe to say that some more work is still needed to firmly establish the string-theory value of L_{min} .

Having clarified that a relation of type (18) is a rather common prediction of theoretical work on quantum gravity, it is then natural to wonder whether such a relation is suggestive of stochastic distance fluctuations of a type that could significantly affect the noise levels of an interferometer. As mentioned, relations such as (18) do not necessarily encode any fuzziness; for example, relation (18) could simply emerge from a theory based on a lattice of points with spacing L_{min} and equipped with a measurement theory consistent with (18). The concept of distance in such a theory would not necessarily be affected by the type of stochastic processes that lead to noise in an interferometer. However, if one does take as guidance the Wheeler-Hawking intuition on space-time foam it makes sense to assume that relation (18) might encode the net effect of some underlying physical processes of the type one would qualify as quantum space-time fluctuations. This (however preliminary) network of intuitions suggests that (18) could be the result of fuzziness for distances D of the type associated with stochastic fluctuations with root-mean-square deviation σ_D given by

$$\sigma_D \sim L_{min} . \quad (20)$$

The associated displacement amplitude spectral density $S_{min}(f)$ should roughly have a $1/\sqrt{f}$ behaviour

$$S_{min}(f) \sim \frac{L_{min}}{\sqrt{f}} , \quad (21)$$

which (using notation set up in Section 4) can be concisely described stating that $L_{min} \sim \mathcal{L}_{\beta=1/2}$. Eq. (21) can be justified using the general relation (4). Substituting the $S_{min}(f)$ of Eq. (21) for the $S(f)$ of Eq. (4) one obtains a σ that approximates the σ_D of Eq. (20) up to small (logarithmic) T_{obs} -dependent corrections. A more detailed description of the displacement amplitude spectral density associated with Eq. (20) can be found in Refs. [90, 91]. For the objectives of these lectures the rough estimate (21) is sufficient since, if indeed $L_{min} \sim L_p$, from (21) one obtains $S_{min}(f) \sim 10^{-35} m/\sqrt{f}$, which is still very far from the sensitivity of even the most advanced modern interferometers, and therefore I shall not be concerned with corrections to Eq. (21).

8.2 Random-walk noise motivated by the analysis of a Salecker-Wigner gedanken experiment

Let me now consider a measurability bound which is encountered when taking into account the quantum properties of devices. It is well understood (see, *e.g.*, Refs. [12, 13, 92, 44, 45, 32]) that the combination of the gravitational properties and the quantum properties of devices can have an important role in the analysis of the operative definition of gravitational observables. Since the analyses [30, 33, 31, 89] that led to the proposal of Eq. (18) only treated the devices in a completely idealized manner (assuming that one could ignore any contribution to the uncertainty in the measurement of D due to the gravitational and quantum properties of devices), it is not surprising that analyses taking into account the gravitational and quantum properties of devices found more significant limitations to the measurability of distances.

Actually, by ignoring the way in which the gravitational properties and the quantum properties of devices combine in measurements of geometry-related physical properties of a system one misses some of the fundamental elements of novelty we should expect for the interplay of gravitation and quantum mechanics; in fact, one would be missing an element of novelty which is deeply associated to the Equivalence Principle. In measurements of physical properties which are not geometry-related one can safely resort to an idealized description of devices. For example, in the famous Bohr-Rosenfeld analysis [93] of the measurability of the electromagnetic field it was shown that the accuracy allowed by the formalism of ordinary quantum mechanics could only be achieved using idealized test particles with vanishing ratio between electric charge and inertial mass. Attempts to generalize the Bohr-Rosenfeld analysis to the study of gravitational fields (see, *e.g.*, Ref. [92]) are of course confronted with the fact that the ratio between gravitational “charge” (mass) and inertial mass is fixed by the Equivalence Principle. While ideal devices with vanishing ratio between electric charge and inertial mass can be considered at least in principle, devices with vanishing ratio between gravitational mass and inertial mass are not admissible in any (however formal) limit of the laws of gravitation. This observation provides one of the strongest elements in support of the idea [13] that the mechanics on which quantum gravity is based must not be exactly the one of ordinary quantum mechanics, since it should accommodate a somewhat different relationship between “system” and “measuring apparatus” and should not rely on the idealized “measuring apparatus” which plays such a central role in the mechanics laws of ordinary quantum mechanics (see, *e.g.*, the “Copenhagen interpretation”).

In trying to develop some intuition for the type of fuzziness that could affect the concept of distance in quantum gravity, it might be useful to consider the way in which the interplay between the gravitational and the quantum properties of devices affects the measurability of distances. In Refs. [12, 13] I have argued¹⁷ that a natural starting point for this type of analysis is provided by the procedure for the measurement of distances which was discussed in influential work¹⁸ by Salecker and Wigner [94]. These authors “measured” (in the “*gedanken*” sense) the distance D between two bodies by exchanging a light signal between them. The

¹⁷I shall comment later in these notes on the measurability analysis reported in Ref. [45], which also took as starting point the mentioned work by Salecker and Wigner, but advocated a different viewpoint and reached different conclusions.

¹⁸The classic Salecker-Wigner work [94] is criticized in the recent paper [54]. As I explain in detail in Ref. [53], the analysis reported in Ref. [54] is incorrect. Whereas Salecker and Wigner sought an operative definition of distances suitable for the Planck regime, the analysis in Ref. [54] relies on several assumptions that appear to be natural in the context of most present-day experiments but are not even meaningful in the Planck regime. Moreover, contrary to the claim made in Ref. [54], the source of $\sqrt{T_{obs}}$ -uncertainty used in the Salecker-Wigner derivation cannot be truly eliminated; unsurprisingly, it can only be traded [53] for another comparable contribution to the total uncertainty in the measurement. In addition to this incorrect criticism of the limit derived by Salecker and Wigner, Ref. [54] also misrepresented the role of the Salecker-Wigner limit in providing motivation for the interferometric studies here considered (and originally proposed

measurement procedure requires *attaching*¹⁹ a light-gun (*i.e.* a device capable of sending a light signal when triggered), a detector and a clock to one of the two bodies and *attaching* a mirror to the other body. By measuring the time T_{obs} (time of observation) needed by the light signal for a two-way journey between the bodies one also obtains a measurement of the distance D . For example, in flat space and neglecting quantum effects one simply finds that $D = cT_{obs}/2$. Within this setup it is easy to realize that the interplay between the gravitational and the quantum properties of devices leads to an irreducible contribution to the uncertainty δD . In order to see this it is sufficient to consider the contribution to δD coming from the uncertainties that affect the motion of the center of mass of the system composed by the light-gun, the detector and the clock. Denoting with x^* and v^* the position and the velocity of the center of mass of this composite device relative to the position of the body to which it is *attached*, and assuming that the experimentalists prepare this device in a state characterised by uncertainties δx^* and δv^* , one easily finds [94, 13]

$$\delta D \geq \delta x^* + T_{obs}\delta v^* \geq \delta x^* + \left(\frac{1}{M_b} + \frac{1}{M_d}\right) \frac{\hbar T_{obs}}{2\delta x^*} \geq \sqrt{\frac{\hbar T_{obs}}{2} \frac{1}{M_d}}, \quad (22)$$

where M_b is the mass of the body, M_d is the total mass of the device composed of the light-gun, the detector, and the clock, and I also used the fact that Heisenberg's *Uncertainty Principle* implies $\delta x^*\delta v^* \geq (1/M_b + 1/M_d)\hbar/2$. [The *reduced mass* $(1/M_b + 1/M_d)^{-1}$ is relevant for the relative motion.] Clearly, from (22) it follows that in order to reduce the contribution to the uncertainty coming from the quantum properties of the devices it is necessary to take the formal “classical-device limit,” *i.e.* the limit²⁰ of infinitely large M_d .

Up to this point I have not yet taken into account the gravitational properties of the devices and in fact the “classical-device limit” encountered above is fully consistent with the

in Refs. [7, 24]): the reader could come out of reading Ref. [54] with the impression that such interferometry-based tests would only be sensitive to quantum-gravity ideas motivated by the Salecker-Wigner limit. As emphasized in Sections 4 and 8 of these notes (and in Ref. [24]) motivation for this phenomenological programme also comes from a long tradition of ideas (developing independently of the ideas related to the Salecker-Wigner limit) on foamy/fuzzy space-time, and from recent work on the possibility that quantum-gravity might induce a deformation of the dispersion relation that characterizes the propagation of the massless particles used as space-time probes in the operative definition of distances. This is already quite clear at least to a portion of the community; for example, in recent work [84] on foamy space-times (without any reference to the Salecker-Wigner related literature) the type of modern-interferometer sensitivity exposed in Refs. [7, 24] was used to constrain certain novel candidate quantum-gravity effects.

¹⁹Of course, for consistency with causality, in such contexts one assumes devices to be “attached non-rigidly,” and, in particular, the relative position and velocity of their centers of mass continue to satisfy the standard uncertainty relations of quantum mechanics.

²⁰A body of finite mass can acquire a nearly-classical behaviour when immersed in a suitable environment (environment-induced decoherence). However, one of the central hypothesis of the work of Salecker and Wigner and followers is that the quantum properties of devices should not be negligible in quantum gravity, and that in particular the in-principle operative definition of distances (which we expect to lie at the foundations of quantum gravity) should not rely on environment-induced decoherence. It appears worth exploring the implications of this hypothesis not only because quantum gravity could be a truly fundamental theory (rather than the effective large-distance description of a more fundamental theory) but also because the operative definition of distances in quantum gravity should be applicable all the way down to the Planck length. It is even plausible [94, 95] that quantum gravity should accommodate an operative definition of a material reference system composed of a network of free-falling particles with relative distances comparable to the Planck length. Within the framework of these intuitions it is indeed quite hard to imagine a decoherence-inducing environment suitable for the in-principle operative definition of distances in quantum gravity. As emphasized in Ref. [53], the analysis reported in Ref. [54] missed this important conceptual element of the Salecker-Wigner approach.

laws of ordinary quantum mechanics. From a physical/phenomenological and conceptual viewpoint it is well understood that the formalism of quantum mechanics is only appropriate for the description of the results of measurements performed by classical devices. It is therefore not surprising that the classical-device (infinite-mass) limit turns out to be required in order to match the prediction $\min \delta D = 0$ of ordinary quantum mechanics.

If one also takes into account the gravitational properties of the devices, a conflict with ordinary quantum mechanics immediately arises because the classical-device (infinite-mass) limit is in principle inadmissible for measurements concerning gravitational effects.²¹ As the devices get more and more massive they increasingly disturb the gravitational/geometrical observables, and well before reaching the infinite-mass limit the procedures for the measurement of gravitational observables cannot be meaningfully performed [12, 13, 45]. In the Salecker-Wigner measurement procedure the limit $M_d \rightarrow \infty$ is not admissible when gravitational interactions are taken into account. At the very least the value of M_d is limited by the requirement that the apparatus should not turn into a black hole (which would not allow the exchange of signals required by the measurement procedure).

These observations render unavoidable the $\sqrt{T_{obs}}$ -dependence of Eq. (22). It is important to realize that this $\sqrt{T_{obs}}$ -dependence of the bound on the measurability of distances comes simply from combining elements of quantum mechanics with elements of classical gravity. As it stands it is not to be interpreted as a quantum-gravity effect. However, as clarified in the opening of this section, if one is interested in modeling properties of the space-time foam it is natural to explore the possibility that the foam be such that distances be affected by stochastic fluctuations with this typical $\sqrt{T_{obs}}$ -dependence. The logic is here the one of observing that stochastic fluctuations associated to the foam would anyway lead to some form of dependence on T_{obs} and in guessing the specific form of this dependence the measurability analysis reviewed in this subsection can be seen as providing motivation for a $\sqrt{T_{obs}}$ -dependence. From this point of view the measurability analysis reviewed in this subsection provides additional motivation for the study of random-walk-type models of distance fuzziness, whose fundamental stochastic fluctuations are characterized (as already discussed in Section 4) by root-mean-square deviation σ_D given by²²

$$\sigma_D \sim \sqrt{L_{QG} c T_{obs}} \quad (23)$$

and by displacement amplitude spectral density $S(f)$ given by

$$S(f) = f^{-1} \sqrt{L_{QG} c} . \quad (24)$$

²¹This conflict between the infinite-mass classical-device limit (which is implicit in the applications of the formalism of ordinary quantum mechanics to the description of the outcome of experiments) and the nature of gravitational interactions has not been addressed within any of the most popular quantum gravity approaches, including critical superstrings [38, 39] and canonical/loop quantum gravity [40]. In a sense somewhat similar to the one appropriate for Hawking's work on black holes [96], this "classical-device paradox" appears to provide an obstruction [13] for the use of the ordinary formalism of quantum mechanics for a description of quantum gravity.

²²As discussed in Refs. [12, 13, 24], this form of σ_D also implies that in quantum gravity any measurement that monitors a distance D for a time T_{obs} is affected by an uncertainty $\delta D \geq \sqrt{L_{QG} c T_{obs}}$. This must be seen as a minimum uncertainty that takes only into account the quantum and gravitational properties of the measuring apparatus. Of course, an even tighter bound can emerge when taking into account also the quantum and gravitational properties of the system under observation. According to the estimates provided in Refs. [30, 33] the contribution to the uncertainty coming from the system is of the type $\delta D \geq L_p$, so that the total contribution (summing the system and the apparatus contributions) might be of the type $\delta D \geq L_p + \sqrt{L_{QG} c T_{obs}}$.

Here the scale L_{QG} plays exactly the same role as in Section 4 (in particular $L_{QG} \equiv \mathcal{L}_{\beta=1}$ in the sense of Section 4). However, seeing L_{QG} as the result of Planck-length fluctuations occurring at a rate of one per Planck time can suggest $L_{QG} \sim L_p$, whereas the different intuition which has gone into the emergence of L_{QG} in this subsection leaves room for different predictions. As already emphasized, by mixing elements of quantum mechanics and elements of gravitation one can only conclude that there could be some $\sqrt{T_{obs}}$ -dependent irreducible contribution to the uncertainty in the measurement of distances. One can then guess that space-time foam might reflect this $\sqrt{T_{obs}}$ -dependence and one can parametrize our ignorance by introducing L_{QG} in the formula $\sqrt{L_{QG} c T_{obs}}$. Within such an argument the estimate $L_{QG} \sim L_p$ could only be motivated on dimensional grounds (L_p is the only length scale available), but there is no direct estimate of L_{QG} within the argument advocated in this subsection. We only have (in the specific sense intended above) a lower limit on L_{QG} which is set by the bare analysis using straightforward combination of elements of ordinary quantum mechanics and elements of ordinary gravity. As seen above, this lower limit on L_{QG} is set by the minimum allowed value of $1/M_d$. Our intuition for L_{QG} might benefit from trying to establish this minimum allowed value of $1/M_d$. As mentioned, a conservative (possibly very conservative) estimate of this minimum value can be obtained by enforcing that M_d be at least sufficiently small to avoid black hole formation. In leading order (*e.g.*, assuming corresponding spherical symmetries) this amounts to the requirement that $M_d < \hbar S_d / (c L_p^2)$, where the length S_d characterizes the size of the region of space where the matter distribution associated to M_d is localized. This observation implies

$$\frac{1}{M_d} > \frac{c L_p^2}{\hbar} \frac{1}{S_d}, \quad (25)$$

which in turn suggests [12] that $L_{QG} \sim \min[L_p^2/S_d]$:

$$\delta D \geq \min \sqrt{\frac{1}{S_d} \frac{L_p^2 c T_{obs}}{2}}. \quad (26)$$

Of course, this estimate is very preliminary since a full quantum gravity would be needed here; in particular, the way in which black holes were handled in my argument might have missed important properties which would become clear only once we have the correct theory. However, it is nevertheless striking to observe that the guess $L_{QG} \sim L_p$ appears to be very high with respect to the lower limit on L_{QG} which we are finding from this estimate; in fact, $L_{QG} \sim L_p$ would correspond to the maximum admissible value of S_d being of order L_p . Since my analysis only holds for devices that can be treated as approximately rigid²³ and any non-rigidity could introduce additional contributions to the uncertainties, it is reasonable

²³The fact that I have included only one contribution from the quantum properties of the devices, the one associated with the quantum properties of the motion of the center of mass, implicitly relies on the assumption that the devices and the bodies can be treated as approximately rigid. Any non-rigidity of the devices could introduce additional contributions to the uncertainty in the measurement of D . This is particularly clear in the case of detector screens and mirrors, whose shape plays a central role in data analysis. Uncertainties in the shape (the relative position of different small parts) of a detector screen or of a mirror would lead to uncertainties in the measured quantity. For large devices some level of non-rigidity appears to be inevitable in quantum gravity. Causality alone (without any quantum mechanics) forbids rigid attachment of two bodies (*e.g.*, two small parts of a device), but is still consistent with rigid motion (bodies are not really attached but because of fine-tuned initial conditions their relative position and relative orientation are constants of motion). When Heisenberg's *Uncertainty Principle* is introduced rigid motion becomes possible only for bodies of infinite mass (otherwise the relative motion inevitably has

to assume that $\max[S_d]$ be some small length (small enough that any non-rigidity would negligibly affect the measurement procedure), but it appears unlikely that $\max[S_d] \sim L_p$. This observation might provide some encouragement for values of L_{QG} smaller than L_p , which after all is the only way to obtain random-walk models consistent with the data analysis reported in Refs. [7, 24].

Later in this section I will consider a particular class of estimates for $\max[S_d]$: if the correct quantum gravity is such that something like (26) holds but with $\max[S_d]$ that depends on δD and/or T_{obs} , one would have a different T_{obs} -dependence (and corresponding f -dependence), as I shall show in one example.

8.3 Random-walk noise motivated by linear deformation of dispersion relations

Besides the analysis of the Salecker-Wigner measurement procedure also the mentioned possibility of quantum-gravity-induced deformation of dispersion relations [5, 46, 41, 21, 27] would be consistent with the idea of random-walk distance fuzziness. The sense in which this is true is clarified by the arguments that follow.

Let me start by going back to the general relation (already discussed in Section 2):

$$c^2 \mathbf{p}^2 \simeq E^2 \left[1 + \xi \left(\frac{E}{E_{QG}} \right)^\alpha \right]. \quad (27)$$

Scenarios (27) with $\alpha = 1$ are consistent with random-walk noise, in the sense that an experiment involving as a device (as a probe) a massless particle satisfying the dispersion relation (27) with $\alpha = 1$ would be naturally affected by a device-induced uncertainty that grows with $\sqrt{T_{obs}}$. From the deformed dispersion relation (27) one is led to energy-dependent velocities [24]

$$v \simeq c \left[1 - \left(\frac{1 + \alpha}{2} \right) \xi \left(\frac{E}{E_{QG}} \right)^\alpha \right], \quad (28)$$

and consequently when a time T_{obs} has lapsed from the moment in which the observer (experimentalist) set off the measurement procedure the uncertainty in the position of the massless probe is given by

$$\delta x \simeq c \delta t + \delta v T_{obs} \simeq c \delta t + \frac{1 + \alpha}{2} \alpha \frac{E^{\alpha-1} \delta E}{E_{QG}^\alpha} c T_{obs}, \quad (29)$$

where δt is the uncertainty in the time of emission of the probe, δv is the uncertainty in the velocity of the probe, δE is the uncertainty in the energy of the probe, and I used the relation between δv and δE that follows from (28). Since the uncertainty in the time of

some irreducible uncertainty). Rigid devices are still available in ordinary quantum mechanics but they are peculiar devices, with infinite mass. [Alternatively, in ordinary quantum mechanics one can take a less fundamental viewpoint on measurement (which does not appear to be natural in the Planck regime [53]) in which the trajectory of the different components/parts of a device are classical because the device is immersed in a decoherence-inducing environment.] When both gravitation and quantum mechanics are introduced rigid devices are no longer available since the infinite-mass limit is then inconsistent with the nature of gravitational devices.

emission of a particle and the uncertainty in its energy are related²⁴ by $\delta t \delta E \geq \hbar$, Eq. (29) can be turned into an absolute bound on the uncertainty in the position of the massless probe when a time T_{obs} has lapsed from the moment in which the observer set off the measurement procedure [24]

$$\delta x \geq c \frac{\hbar}{\delta E} + \frac{1+\alpha}{2} \alpha \frac{E^{\alpha-1} \delta E}{E_{QG}^\alpha} T_{obs} \geq \sqrt{\left(\frac{\alpha + \alpha^2}{2}\right) \left(\frac{E}{E_{QG}}\right)^{\alpha-1} \frac{c^2 \hbar T_{obs}}{E_{QG}}}. \quad (30)$$

For $\alpha = 1$ the E -dependence on the right-hand side of Eq. (30) disappears and one is led again to a δx of the type (*constant*) $\cdot \sqrt{T_{obs}}$:

$$\delta x \geq \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}}. \quad (31)$$

When massless probes are used in the measurement of a distance D the uncertainty (31) in the position of the probe translates directly into an uncertainty on D :

$$\delta D \geq \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}}. \quad (32)$$

This was already observed in Refs. [46, 23, 27] which considered the implications of deformed dispersion relations (27) with $\alpha = 1$ for the operative definition of distances.

Since deformed dispersion relations (27) with $\alpha = 1$ have led us to the same measurability bound already encountered both in the analysis of the Salecker-Wigner measurement procedure and the analysis of simple-minded random-walk models of quantum space-time fluctuations, if we assume again that such measurability bounds emerge in a full quantum gravity as a result of corresponding quantum fluctuations (fuzziness), we are led once again to random-walk noise:

$$\sigma_D \sim \sqrt{\frac{c^2 \hbar T_{obs}}{E_{QG}}}. \quad (33)$$

8.4 Noise motivated by quadratic deformation of dispersion relations

In the preceding subsection I observed that quantum-gravity deformed dispersion relations (27) with $\alpha = 1$ can also motivate random-walk noise $\sigma_D \sim (\text{constant}) \cdot \sqrt{T_{obs}}$. If we use the same line of reasoning that connects a measurability bound to a scenario for fuzziness when $\alpha \neq 1$ we appear to find $\sigma_D \sim \mathcal{G}(E/E_{QG}) \cdot \sqrt{T_{obs}}$, where $\mathcal{G}(E/E_{QG})$ is a (α -dependent) function of E/E_{QG} . However, in these cases with $\alpha \neq 1$ clearly the connection between measurability bound and fuzzy-distance scenario cannot be this simple; in fact, the energy of the probe E which naturally plays a role in the context of the derivation of the measurability

²⁴It is well understood that the $\delta t \delta E \geq \hbar$ relation is valid only in a weaker sense than, say, Heisenberg's Uncertainty Principle $\delta x \delta p \geq \hbar$. This has its roots in the fact that the time appearing in quantum-mechanics equations is just a parameter (not an operator), and in general there is no self-adjoint operator canonically conjugate to the total energy, if the energy spectrum is bounded from below [97, 55]. However, $\delta t \delta E \geq \hbar$ does relate δt intended as uncertainty in the time of emission of a particle and δE intended as uncertainty in the energy of that same particle.

bound does not have an obvious counter-part in the context of the conjectured fuzzy-distance scenario.

In order to preserve the conjectured connection between measurability bounds and fuzzy-distance scenarios one can be tempted to envision that if $\alpha \neq 1$ the interferometer noise levels induced by space-time fuzziness might be of the type [24]

$$\sigma_D \sim \sqrt{\left(\frac{\alpha + \alpha^2}{2}\right) \left(\frac{E^*}{E_{QG}}\right)^{\alpha-1} \frac{c^2 \hbar T_{obs}}{E_{QG}}}, \quad (34)$$

where E^* is some energy scale characterizing the physical context under consideration. [For example, at the intuitive level one might conjecture that E^* could characterize some sort of energy density associated with quantum fluctuations of space-time or an energy scale associated with the masses of the devices used in the measurement process.]

Since $\alpha \geq 1$ in all quantum-gravity approaches believed to support deformed dispersion relations, it appears likely that the factor $(E^*/E_{QG})^{\alpha-1}$ would suppress the random-walk noise effect in all contexts with $E^* < E_{QG}$. Besides the case $\alpha = 1$ (linear deformation) also the case $\alpha = 2$ (quadratic deformation) deserves special interest since it can emerge quite naturally in quantum-gravity approaches (see, *e.g.*, Ref. [22]).

8.5 Noise with $f^{-5/6}$ amplitude spectral density

In Subsection 8.2 a bound on the measurability of distances based on the Salecker-Wigner procedure was used as additional motivation for experimental tests of interferometer noise of random-walk type, with f^{-1} amplitude spectral density and $\sqrt{T_{obs}}$ root-mean-square deviation. In this subsection I shall pursue further the observation that the relevant measurability bound could be derived by simply insisting that the devices do not turn into black holes. That observation allowed to derive Eq. (26), which expresses the minimum uncertainty δD on the measurement of a distance D (*i.e.* the measurability bound for D) as proportional to $\sqrt{T_{obs}}$ and $\sqrt{1/S_d}$. Within that derivation the minimum uncertainty is obtained in correspondence of $\max[S_d]$, the maximum value of S_d consistent with the structure of the measurement procedure. I was therefore led to consider how large S_d could be while still allowing to disregard any non-rigidity in the quantum motion of the device (which could introduce additional contributions to the uncertainties). Something suggestive of the random-walk noise scenario emerged by simply assuming that $\max[S_d]$ be independent of T_{obs} and independent of the accuracy δD that the observer would wish to achieve. However, as mentioned, the same physical intuition that motivates some of the fuzzy space-time scenarios here considered also suggests that quantum gravity might require a novel measurement theory, possibly involving a new type of relation between system and measuring apparatus. Based on this intuition, it seems reasonable to contemplate the possibility that $\max[S_d]$ might actually depend on δD .

It is such a scenario that I want to consider in this subsection. In particular I want to consider the case $\max[S_d] \sim \delta D$, which, besides being simple, has the plausible property that it allows only small devices if the uncertainty to be achieved is small, while it would allow correspondingly larger devices if the observer was content with a larger uncertainty. This is also consistent with the idea that elements of non-rigidity in the quantum motion of extended devices could be neglected if anyway the measurement is not aiming for great accuracy, while they might even lead to the most significant contributions to the uncertainty if all other sources of uncertainty are very small. [Salecker and Wigner [94] would also argue that “large” devices are not suitable for very accurate space-time measurements (they end up being “in the way” of the measurement procedure) while they might be admissible if space-time is being probed rather softly.]

In this scenario with $\max[S_d] \sim \delta D$, Eq. (26) takes the form

$$\delta D \geq \sqrt{\frac{1}{S_d} \frac{L_p^2 c T_{obs}}{2}} \geq \sqrt{\frac{L_p^2 c T_{obs}}{2 \delta D}}, \quad (35)$$

which actually gives

$$\delta D \geq \left(\frac{1}{2} L_p^2 c T_{obs} \right)^{1/3}. \quad (36)$$

As done with the other measurability bounds, I have proposed [7, 24] to take Eq. (36) as motivation for the investigation of a corresponding fuzziness scenario characterised by

$$\sigma_D \sim \left(\tilde{L}_{QG}^2 c T_{obs} \right)^{1/3}. \quad (37)$$

Notice that in this equation I replaced L_p with a generic length scale \tilde{L}_{QG} , since it is possible that the heuristic argument leading to Eq. (37) might have captured the qualitative structure of the phenomenon while providing an incorrect estimate of the relevant length scale. Also notice that Eq. (36) has the same form as the relations emerged in other measurability analyses [45, 43], even though those analyses adopted a very different viewpoint (and even the physical interpretation of the elements of Eq. (36) was different, as explained in the next section).

As observed in Refs. [7, 24] the $T_{obs}^{1/3}$ dependence of σ_D is associated with displacement amplitude spectral density with $f^{-5/6}$ behaviour:

$$\mathcal{S}(f) = f^{-5/6} (\tilde{L}_{QG}^2 c)^{1/3}. \quad (38)$$

Therefore the measurability analyses discussed in this subsection provides motivation for the investigation of the case $\beta = 5/6$ (using again the notation set up in Section 4).

9 ABSOLUTE MEASURABILITY BOUND FOR THE AMPLITUDE OF A GRAVITY WAVE

The bulk of this Article (presented in the previous three sections) concerns the implications of distance fuzziness for interferometry. Various scenarios for distance fuzziness were motivated either by a general Wheeler-Hawking-inspired phenomenological parametrization or by intuitive arguments based on the possibility of quantum-gravity-induced deformations of dispersion relations or quantum-gravity²⁵ distance-measurability analyses within

²⁵My observations within the Salecker-Wigner setup do pertain to the quantum-gravity realm because I took into account the gravitational properties of the devices and I also, like Salecker and Wigner, removed the assumption of classicality of the devices. If one was only putting together some properties of gravitation and quantum mechanics one could at best probe a simple limiting behaviour of quantum gravity, but by removing one of the conceptual ingredients of ordinary quantum mechanics it is plausible that we get a glimpse of a true property of quantum gravity. The Salecker-Wigner study [94] (just like the Bohr-Rosenfeld analysis [93]) suggests that among the conceptual elements of quantum mechanics the one that is most likely (although there are of course no guarantees) to succumb to the unification of gravitation and quantum mechanics is the requirement for devices to be treated as classical.

the Salecker-Wigner setup. My observation that distance fuzziness would be felt by interferometers as a fundamental additional source of noise (*i.e.* as some sort of fundamental source of stochastic gravity-wave background) also implies that, if indeed quantum gravity hosts distance fuzziness, there would be a quantum-gravity induced bound on the measurability of gravity waves. This section is parenthetical, within the logical line of this Article, in the sense that I will assume in this section that there is no distance fuzziness. The objective is one of showing that even without distance fuzziness it appears that the measurability of gravity waves should be limited in quantum gravity.

The strategy I will use to derive this bound is an adaptation of the Salecker-Wigner framework to the analysis of gravity-wave measurability. Basically, while the Salecker-Wigner framework concerns the measurement of a distance D , I shall here apply the same reasoning to the measurement of “distance displacements” in interferometers arms (of length L) of the type that could be induced by a gravity wave.

Having clarified in which sense this section represents a deviation from the main bulk of observations reported in the present Article, let me start the discussion by reminding the reader of the fact that, as already mentioned in Section 2, the interference pattern generated by a modern interferometer can be remarkably sensitive to changes in the positions of the mirrors relative to the beam splitter, and is therefore sensitive to gravitational waves (which, as described in the *proper reference frame* [47], have the effect of changing these relative positions). With just a few lines of simple algebra one can show that an ideal gravitational wave of amplitude h and reduced²⁶ wavelength λ_{gw}^o propagating along the direction orthogonal to the plane of the interferometer would cause a change in the interference pattern as for a phase shift of magnitude $\Delta\phi = D_L/\lambda^o$, where λ^o is the reduced wavelength of the laser beam used in the measurement procedure and [47, 98]

$$D_L \sim 2 h \lambda_{gw}^o \left| \sin \left(\frac{L}{2\lambda_{gw}^o} \right) \right| , \quad (39)$$

is the magnitude of the change caused by the gravitational wave in the length of the arms of the interferometer. (The changes in the lengths of the two arms have opposite sign [47].)

As already mentioned in Section 2, modern techniques allow to construct gravity-wave interferometers with truly remarkable sensitivity; in particular, at least for gravitational waves with λ_{gw}^o of order $10^3 Km$, the next LIGO/VIRGO generation of detectors should be sensitive to h as low as $3 \cdot 10^{-22}$. Since $h \sim 3 \cdot 10^{-22}$ causes a D_L of order $10^{-18}m$ in arms lengths L of order $3Km$, it is not surprising that in the analysis of gravity-wave interferometers, in spite of their huge size, one ends up having to take into account [47] the type of quantum effects usually significant only for the study of processes at or below the atomic scale. In particular, there is the so-called *standard quantum limit* on the measurability of h that results from the combined minimization of *photon shot noise* and *radiation pressure noise*. While a careful discussion of these two noise sources (which the interested reader can find in Ref. [47]) is quite insightful, here I shall rederive this *standard quantum limit* in an alternative²⁷ and straightforward manner (also discussed in Ref. [98]), which relies on the

²⁶I report these results in terms of reduced wavelengths λ^o (which are related to the wavelengths λ by $\lambda^o = \lambda/(2\pi)$) in order to avoid cumbersome factors of π in some of the formulas.

²⁷While the *standard quantum limit* can be equivalently obtained either from the combined minimization of *photon shot noise* and *radiation pressure noise* or from the application of Heisenberg’s uncertainty principle to the position and momentum of the mirror, it is this author’s opinion that there might actually be a fundamental difference between the two derivations. In fact, it appears (see, *e.g.*, Ref. [90] and references therein) that the limit obtained through combined minimization of *photon shot noise* and *radiation pressure noise* can be violated by careful exploitation of the properties of squeezed light, whereas the limit obtained through the application of Heisenberg’s uncertainty principle to the position and momentum of the mirror is truly fundamental.

application of Heisenberg's uncertainty principle to the position and momentum of a mirror relative to the position of the beam splitter. This can be done along the lines of my analysis of the Salecker-Wigner procedure for the measurement of distances. Since the mirrors and the beam splitter are macroscopic, and therefore the corresponding momenta and velocities are related non-relativistically, Heisenberg's uncertainty principle implies that ²⁸

$$\delta x \delta v \geq \frac{\hbar}{2} \left(\frac{1}{M_m} + \frac{1}{M_b} \right) \geq \frac{\hbar}{2M_m}, \quad (40)$$

where δx and δv are the uncertainties in the relative position and relative velocity, M_m is the mass of the mirror, M_b is the mass of the beam splitter. [Again, the relative motion is characterised by the *reduced mass*, which is given in this case by $(1/M_m + 1/M_b)^{-1}$.]

Clearly, the high precision of the planned measurements requires [47, 98] that the position of the mirrors be kept under control during the whole time $2L/c$ that the beam spends in between the arms of the detector before superposition. When combined with (40) this leads to the finding that, for any given value of M_m , the D_L induced by the gravitational wave can be measured only up to an irreducible uncertainty, the so-called *standard quantum limit*:

$$\delta D_L \geq \delta x + \delta v 2 \frac{L}{c} \geq \delta x + \frac{\hbar L}{c M_m \delta x} \geq \sqrt{\frac{\hbar L}{c M_m}}. \quad (41)$$

The case of gravity-wave measurements is a canonical example of my general argument that the infinite-mass classical-device limit underlying ordinary quantum mechanics is inconsistent with the nature of gravitational measurements. As the devices get more and more massive they not only increasingly disturb the gravitational/geometrical observables, but eventually (well before reaching the infinite-mass limit) they also render impossible [12, 13] the completion of the procedure of measurement of gravitational observables. In trying to assess how this observation affects the measurability of the properties of a gravity wave let me start by combining Eqs. (39) and (41):

$$\delta h = \delta \left(\frac{D_L}{L} \right) = h \frac{\delta D_L}{D_L} \geq \frac{\sqrt{\frac{\hbar L}{c M_m}}}{2 \lambda_{gw}^o \left| \sin \left(\frac{L}{2 \lambda_{gw}^o} \right) \right|}. \quad (42)$$

In complete analogy with some of the observations made in Section 3 concerning the measurability of distances, I observe that, when gravitational effects are taken into account, the limit of infinite mirror mass is of course inadmissible. At the very least M_m must be small enough that the mirror does not turn into a black hole.²⁹ In order for the mirror not to be a black hole one requires $M_m < \hbar S_m / (c L_p^2)$, where S_m is the *size* of the region of space occupied by the mirror. This observation combined with (42) implies that one would have obtained a bound on the measurability of h if one found a maximum allowed mirror size S_m . In estimating this maximum S_m one can be easily led to some extreme and incorrect assumptions. In particular, one could suppose that in order to achieve a sensitivity to D_L as low as $10^{-18}m$ it might be necessary to “accurately position” each $10^{-36}m^2$ surface element of the mirror. If this was really necessary, our line of argument would then lead to a rather large

²⁸Note that in the setup of gravity-wave interferometers the test masses are required to be free-falling [47]. In such a context the type of observations reported in Ref. [54] is not only inadequate for in-principle analyses of measurability in the full quantum-gravity regime but in most cases, as a result of the free-fall requirement, it will also be inapplicable in the ordinary context of present-day interferometers.

²⁹This is of course a very conservative bound, since a mirror stops being useful as a device well before it turns into a black hole, but even this conservative approach leads to an interesting conclusion.

measurability bound. Fortunately, the phase of the wavefront of the reflected light beam is determined by the average position of all the atoms across the beam's width, and microscopic irregularities in the structure of the mirror only lead to scattering of a small fraction of light out of the beam. This suggests that in our analysis the size of the mirror should be assumed to be of the order of the width of the beam [47]. So S_m cannot be too small, but on the other hand in light of this observation, and taking into account the in-principle nature³⁰ of the analysis I am performing, it is clear that S_m could not be too large either, and in particular it appears safe to assume that S_m should be smaller than the λ_{gw}^o of the gravity wave which one is planning to observe. If S_m is indeed the width of the beam (and therefore the effective size of the mirror), then one must exclude the possibility $S_m > \lambda_{gw}^o$ because otherwise the same gravity wave which one is intending to observe would cause phenomena preventing the proper completion of the measurement procedure (*e.g.* deforming the mirror and leading to a nonlinear relation between D_L and h). The conservative bound $S_m < \lambda_{gw}^o$ also appears to be safe with respect to the expectations of another type of intuition, usually resulting from experience with table-top interferometers. Within this assumption one is always tempted to think of the mirror as *attached* to a very massive body. Even setting aside the limitations on this type of idealized attachments that are set by the uncertainty principle and causality, it appears that the bound $S_m < \lambda_{gw}^o$ should be safe because of the requirement that the mirror be free-falling. [It actually seems extremely conservative to just demand of such a free-falling interferometer mirror that the sum of its mass and the mass of any body “attached” to it should not exceed the mass of a black hole of size λ_{gw}^o .]

In summary, it looks very safe to assume that M_m should be smaller than $\hbar\lambda_{gw}^o/(cL_p^2)$, and this can be combined with (42) to obtain the measurability bound

$$\delta h > \frac{L_p}{2\lambda_{gw}^o} \frac{\sqrt{L/\lambda_{gw}^o}}{\left| \sin\left(\frac{L}{2\lambda_{gw}^o}\right) \right|}. \quad (43)$$

This result not only sets a lower bound on the measurability of h with given arm's length L , but also encodes an absolute (*i.e.* irrespective of the value of L) lower bound, as a result of the fact that the function $\sqrt{x}/|\sin(x/2)|$ has an absolute minimum: $\min[\sqrt{x}/\sin(x/2)] \sim 1.66$. This novel measurability bound is a significant departure from the principles of ordinary quantum mechanics, especially in light of the fact that it describes a limitation on the measurability of a single observable (the amplitude h of a gravity wave), and that this limitation turns out to depend on the value (not the associated uncertainty) of another observable (the reduced wavelength λ_{gw}^o of the same gravity wave). It is also significant that this new bound (43) encodes an aspect of a novel type of interplay between system and measuring apparatus in quantum-gravity regimes; in fact, in deriving (43) a crucial role was played by the fact that in accurate measurements of gravitational/geometrical observables it is no longer possible [13] to advocate an idealized description of the devices.

Also the T_{obs} -dependent bound on the measurability of distances which I reviewed in Section 3 encodes a departure from ordinary quantum mechanics and a novel type of interplay between system and measuring apparatus, but the bound (43) on the measurability of the amplitude of a gravity wave (which is one of the new results reported in the present Article) should provide even stronger motivation for the search of formalisms in which quantum

³⁰For the gravitational waves to which LIGO/VIRGO will be most sensitive, which have λ_{gw}^o of order $10^3 Km$, the requirement $S_m < \lambda_{gw}^o$ simply states that the size of mirrors should be smaller than $10^3 Km$. This bound might appear very conservative, but I am trying to establish an in-principle limitation on the measurability of h . Since such a bound was not previously established, in this first study I just want to clarify that the bound exists, rather than dwell on the exact magnitude of the bound. I therefore prefer to be very conservative in my estimates.

gravity is based on a new mechanics, not exactly given by ordinary quantum mechanics. In fact, while one might still hope to find alternatives to the Salecker-Wigner measurement procedure that allow to measure distances evading the corresponding measurability bounds, it appears hard to imagine that there could be anything (even among “gedanken laboratories”) better than an interferometer for measurements of the amplitude of a gravity wave.

It is also important to realize that the bound (43) cannot be obtained by just assuming that the Planck length L_p provides the minimum uncertainty for distances (and distance variations). In fact, if the only limitation was $\delta D_L \geq L_p$ the resulting uncertainty on h , which I denote with $\delta h^{(L_p)}$, would have the property

$$\min[\delta h^{(L_p)}] = \min \left[\frac{L_p}{2 \lambda_{gw}^o \left| \sin \left(\frac{L}{2 \lambda_{gw}^o} \right) \right|} \right] = \frac{L_p}{2 \lambda_{gw}^o} , \quad (44)$$

whereas, exploiting the above-mentioned properties of the function $\sqrt{x}/|\sin(x/2)|$, from (43) one finds³¹

$$\min[\delta h] > \min \left[\frac{L_p}{2 \lambda_{gw}^o} \frac{\sqrt{L/\lambda_{gw}^o}}{\left| \sin \left(\frac{L}{2 \lambda_{gw}^o} \right) \right|} \right] > \min[\delta h^{(L_p)}] . \quad (45)$$

In general, the dependence of $\delta h^{(L_p)}$ on λ_{gw}^o is different from the one of δh . Actually, in light of the comparison of (44) with (45) it is amusing to observe that the bound (43) could be seen as the result of a minimum length L_p combined with an λ_{gw}^o -dependent correction. This would be consistent with some of the ideas mentioned in Section 3 (the energy-dependent effect of *in vacuo* dispersion and the corresponding proposal (33) for distance fuzziness) in which the magnitude of the quantum-gravity effect depends rather sensitively on some energy-related aspect of the problem under investigation (just like λ_{gw}^o for the gravity wave).

It is easy to verify that the bound (43), would not observably affect the operation of even the most sophisticated planned interferometers. However, in the spirit of what I did in the previous sections considering the operative definition of distances, also for the amplitudes of gravity waves the fact that we have encountered an obstruction in the measurement analysis based on ordinary quantum mechanics (and the fact that by mixing gravitation and quantum mechanics we have obtained some intuition for novel qualitative features of such gravity-wave amplitudes in quantum gravity) could be used as starting point for the proposal of novel quantum-gravity effects possibly larger than the estimate (43). Although possibly very interesting, these fully quantum-gravity scenarios for the properties of gravity-wave amplitudes will not be explored in these notes. I just want to observe that the strain sensitivity ($S_h(f) \equiv S(f)/L$) of order $10^{-22}/\sqrt{Hz}$ which is soon going to be achieved by several detectors [48, 49, 52, 79] corresponds to a rather natural scale for a fundamental quantum-gravity-induced stochastic-gravity-wave-like noise; in fact, $10^{-22}/\sqrt{Hz} \simeq \sqrt{L_p}/c$.

³¹I am here (for “pedagogical” purposes) somewhat simplifying the comparison between δh and $\delta h^{(L_p)}$. As mentioned, in principle one should take into account both uncertainties inherent in the “system” under observation, which are likely to be characterized exclusively by the Planck-length bound, and uncertainties coming from the “measuring apparatus”, which might easily involve other length (or time) scales besides the Planck length. It would therefore be proper to compare $\delta h^{(L_p)}$, which would be the only contribution present in the conventional idealization of “classical devices”, with the sum $\delta h + \delta h^{(L_p)}$, which, as appropriate for quantum gravity, provides a sum of system-inherent uncertainties plus apparatus-induced uncertainties.

10 RELATIONS WITH OTHER QUANTUM-GRAVITY APPROACHES

In this section I comment on the connections and the differences between some of the ideas that I reviewed in these notes and other quantum-gravity ideas.

10.1 Canonical Quantum Gravity

One of the most popular quantum-gravity approaches is the one in which the ordinary canonical formalism of quantum mechanics is applied to (some formulation of) Einstein's Gravity. In spite of the fact that [13] some of the observations reviewed in the previous sections suggest that quantum gravity should require a new mechanics, not exactly given by ordinary quantum mechanics, it is very interesting³² that some of the phenomena considered in the previous sections have also emerged in studies of canonical quantum gravity.

As mentioned, the most direct connection was found in the study reported in Ref. [41], which was motivated by Ref. [5]. In fact, Ref. [41] shows that the popular canonical/loop quantum gravity [40] admits the phenomenon of deformed dispersion relations, with the deformation going linearly with the Planck length.

Concerning the bounds on the measurability of distances it is probably fair to say that the situation in canonical/loop quantum gravity is not yet clear because the present formulations do not appear to lead to a compelling candidate “length operator.” This author would like to interpret the problems associated with the length operator as an indication that perhaps something unexpected might actually emerge in canonical/loop quantum gravity as a length operator, possibly something with properties fitting the intuition of some of the scenarios for fuzzy distances which I reviewed. Actually, the random-walk space-time fuzziness model might have a (somewhat weak, but intriguing) connection with “quantum mechanics applied to gravity” at least to the level seen by comparison with the scenario discussed in Ref. [100], which was motivated by the intuition that is emerging from investigations of canonical/loop quantum gravity. The “moves” of Ref. [100] share many of the properties of the “random steps” of the random-walk models here considered.

10.2 Critical and non-critical String Theories

Unfortunately, in the popular quantum-gravity approach based on critical superstrings³³ not many results have been derived concerning directly the quantum properties of space-time. Perhaps the most noticeable such results are the ones on limitations on the measurability of distances emerged in the scattering analyses reported in Refs. [31, 89], which I already mentioned.

A rather different picture is emerging (through the difficult technical aspects of this rich formalism) in *Liouville* (non-critical) strings [19], whose development was partly motivated

³²I am here taking a viewpoint that might be summarized rephrasing a comment by B.S. De Witt in Ref. [99]. While some of the arguments reviewed here appear to indicate that ordinary quantum mechanics cannot suffice for quantum gravity, it is still plausible that the language of ordinary quantum mechanics might be a useful tool for the description of its own demise. This would be analogous to something we have learned in the study of special relativity: one could [99] insist on describing the observed Lorentz-Fitzgerald contraction as the result of relativistic modifications in the force law between atoms, but in order to capture the true essence of the new regime it is necessary to embrace the new conceptual framework of special relativity.

³³As already mentioned the mechanics of critical superstrings is just an ordinary quantum mechanics. All of the new structures emerging in this exciting formalism are the result of applying ordinary quantum mechanics to the dynamics of extended fundamental objects, rather than point-like objects (particles).

by intuition concerning the quantum-gravity vacuum that is rather close to the one traditionally associated with the mentioned works of Wheeler and Hawking. Evidence has been found [46] in *Liouville* strings supporting the validity of deformed dispersion relations, with the deformation going linearly with the Planck/string length. In the sense clarified in Subsection 8.3 this approach might also host a bound on the measurability of distances which grows with $\sqrt{T_{obs}}$.

10.3 Other types of measurement analyses

Because of the lack of experimental input, it is not surprising that many authors have been seeking some intuition on quantum gravity by formal analyses of the ways in which the interplay between gravitation and quantum mechanics could affect measurement procedures. A large portion of these analyses produced a “ $\min[\delta D]$ ” with D denoting a distance; however, the same type of notation was used for structures defined in significantly different ways. Also different meanings have been given by different authors to the statement “absolute bound on the measurability of an observable.” Quite important for the topics here discussed are the differences (which might not be totally transparent as a result of this unfortunate choice of overlapping notations) between the approach advocated in Refs. [7, 12, 13, 24] and the approaches advocated in Refs. [94, 44, 45, 43]. In Refs. [7, 12, 13, 24] “ $\min[\delta D]$ ” denotes an absolute limitation on the measurability of a distance D . The studies [94, 44, 43] analyzed the interplay of gravity and quantum mechanics in defining a net of time-like geodesics, and in those studies “ $\min[\delta D]$ ” characterizes the maximum “tightness” achievable for the net of time-like geodesics. Moreover, in Refs. [94, 44, 45, 43] it was required that the measurement procedure should not affect/modify the geometric observable being measured, and “absolute bounds on the measurability” were obtained in this specific sense. Instead, in Refs. [12, 13, 24] it was envisioned that the observable which is being measured might depend also on the devices (the underlying view is that observables in quantum gravity would always be, in a sense, shared properties of “system” and “apparatus”), and it was only required that the nature of the devices be consistent with the various stages of the measurement procedure (for example if a device turned into a black hole some of the exchanges of signals needed for the measurement would be impossible). The measurability bounds of Refs. [12, 13, 24] are therefore to be intended from this more fundamental perspective, and this is crucial for the possibility that these measurability bounds be associated to a fundamental quantum-gravity mechanism for “fuzziness” (quantum fluctuations of space-time). The analyses reported in Refs. [94, 44, 45, 43] did not include any reference to fuzzy space-times of the type operatively defined in terms of stochastic processes in Section 4 (and in Ref. [24]).

The more fundamental nature of the bounds obtained in Refs. [12, 13, 24] is also crucial for the arguments suggesting that quantum gravity might require a new mechanics, not exactly given by ordinary quantum mechanics. The analyses reported in Refs. [94, 44, 45, 43] did not include any reference to this possibility.

Having clarified that there is a “double difference” (different “ \min ” and different “ δD ”) between the meaning of $\min[\delta D]$ adopted in Refs. [7, 12, 13, 24] and the meaning of $\min[\delta D]$ adopted in Refs. [94, 44, 45, 43], it is however important to notice that the studies reported in Refs. [44, 45, 43] were among the first studies which showed how in some aspects of measurement analysis the Planck length might appear together with other length scales in the problem. For example, a quantum-gravity effect naturally involving something of length-squared dimensions might not necessarily go like L_p^2 : in some cases it could go like ΛL_p , with Λ some other length scale in the problem.

Interestingly, the analysis of the interplay of gravity and quantum mechanics in defining a net of time-like geodesics reported in Ref. [44] concluded that the maximum “tightness” achievable for the geodesics would be characterized by $\sqrt{L_p^2 R^{-1} s}$, where R is the radius of the (spherically symmetric) clocks whose world lines define the network of geodesics, and s

is the characteristic distance scale over which one is intending to define such a network. The $\sqrt{L_p^2 R^{-1} s}$ maximum tightness discussed in Ref. [44] is formally analogous to Eq. (26), but, as clarified above, this “maximum tightness” was defined in a very different (“doubly different”) way, and therefore the two proposals have completely different physical implications. Actually, in Ref. [44] it was also stated that for a single geodesic distance (which might be closer to the type of distance measurability analysis reported in Refs. [12, 13, 24]) one could achieve accuracy significantly better than the formula $\sqrt{L_p^2 R^{-1} s}$, which was interpreted in Ref. [44] as a direct result of the structure of a network of geodesics.

Relations of the type $\min[\delta D] \sim (L_p^2 D)^{(1/3)}$, which are formally analogous to Eq. (36), were encountered in the analysis of maximum tightness achievable for a geodesics network reported in Ref. [43] and in the analysis of measurability of distances reported in Ref. [45]. Although once again the definitions of “ \min ” and “ δD ” used in these studies are completely different from the ones relevant for the “ $\min[\delta D]$ ” of Eq. (36).

11 QUANTUM GRAVITY, NO STRINGS (OR LOOPS) ATTACHED

Some of the arguments reviewed in these notes appear to suggest that quantum gravity might require a mechanics not exactly of the type of ordinary quantum mechanics. In particular, the new mechanics might have to accommodate a somewhat different relationship (in a sense, “more democratic”) between “system” and “measuring apparatus”, and should take into account the fact that the limit in which the apparatus behaves classically is not accessible once gravitation is turned on. The fact that the most popular quantum-gravity approaches, including critical superstrings and canonical/loop quantum gravity, are based on ordinary quantum mechanics but seem inconsistent with a correspondence between formalism and measurability bounds of the type sought and found in non-gravitational quantum mechanics (through the work of Bohr, Rosenfeld, Landau, Peierls, Einstein, Salecker, Wigner and many others), represents, in this author’s humble opinion, one of the outstanding problems of these approaches. Still, it is of great importance for quantum-gravity research that these approaches continue to be pursued very aggressively: they might eventually encounter along their development unforeseeable answers to these questions or else, as they are “pushed to the limit”, they might turn out to fail in a way that provides insight on the correct theory. However, the observations pointing us toward deviations from ordinary quantum-mechanics could provide motivation for the parallel development of alternative quantum-gravity approaches. But how could we envision quantum gravity with no strings (or “canonical loops”) attached? More properly, how can we devise a new mechanics when we have no direct experimental data on its structure? Classical mechanics was abandoned for quantum mechanics only after a relatively long period of analysis of physical problems such as black-body spectrum and photoelectric effect which contained very relevant information. We don’t seem to have any such insightful physical problem. At best we might have identified the type of conceptual issues which Mach had discussed with respect to Newtonian physics. It is amusing to notice that the analogy with Machian conceptual analyses might actually be quite proper, since at the beginning of this century we were forced to renounce to the comfort of the reference to “absolute space” and now that we are reaching the end of this century we might be forced to renounce to the comfort of an idealized classical measuring apparatus.

Our task is that much harder in light of the fact that (unless something like large extra dimensions is verified in Nature) we must make a gigantic leap from the energy scales we presently understand to Planckian energy scales. While of course it is not impossible that we eventually do come up with the correct recipe for this gigantic jump, one less optimistic

strategy that might be worth pursuing is the one of trying to come up with some effective theory useful for the description of new space-time-related phenomena occurring in an energy-scale range extending from somewhere not much above presently achievable energies up to somewhere safely below the Planck scale. These theories might provide guidance to experimentalists, and in turn (if confirmed by experiments) might provide a useful intermediate step toward the Planck scale. For those who are not certain that we can make a lucky guess of the whole giant step toward the Planck scale³⁴ this strategy might provide a possibility to eventually get to the Planck regime only after a (long and painful) series of intermediate steps. Some of the ideas discussed in the previous sections can be seen as examples of this strategy. In this section I collect additional relevant material.

11.1 A low-energy effective theory of quantum gravity

While the primary emphasis has been on experimental tests of quantum-gravity-motivated candidate phenomena, some of the arguments (which are based on Refs. [12, 13, 24]) reviewed in these lecture notes can be seen as attempts to identify properties that one could demand of a theory suitable for a first stage of partial unification of gravitation and quantum mechanics. This first stage of partial unification would be a low-energy effective theory capturing only some rough features of quantum gravity. In particular, as discussed in Refs. [23, 13, 24], it is plausible that the most significant implications of quantum gravity for low-energy (large-distance) physics might be associated with the structure of the non-trivial “quantum-gravity vacuum”. A satisfactory picture of this vacuum is not available at present, and therefore we must generically characterize it as the appropriate new concept that in quantum gravity takes the place of the ordinary concept of “empty space”; however, it is plausible that some of the arguments by Wheeler, Hawking and followers, attempting to develop an intuitive description of the quantum-gravity vacuum, might have captured at least some of its actual properties. Therefore the experimental investigations of space-time foam discussed in some of the preceding sections could be quite relevant for the search of a theory describing a first stage of partial unification of gravitation and quantum mechanics.

Other possible elements for the search of such a theory come from studies suggesting that this unification might require a new (non-classical) concept of measuring apparatus and a new relationship between measuring apparatus and system. I have reviewed some of the relevant arguments [12, 13] through the discussion of the Salecker-Wigner setup for the measurement of distances, which manifested the problems associated with the infinite-mass classical-device limit. As mentioned, a similar conclusion was already drawn in the context of attempts (see, *e.g.*, Ref. [92]) to generalize to the study of the measurability of gravitational fields the famous Bohr-Rosenfeld analysis [93] of the measurability of the electromagnetic field. It seems reasonable to explore the possibility that already the first stage of partial unification of gravitation and quantum mechanics might require a new mechanics. A (related) plausible feature of the correct effective low-energy theory of quantum-gravity is (some form of) a novel bound on the measurability of distances. This appears to be an inevitable consequence of relinquishing the idealized methods of measurement analysis that rely on the artifacts of the infinite-mass classical-device limit. If indeed one of these novel measurability bounds holds in the physical world, and if indeed the structure of the quantum-gravity vacuum is non-trivial and involves space-time fuzziness, it appears also plausible that this two features be related, *i.e.* that the fuzziness of space-time would be ultimately responsible for the measurability bounds. It is also plausible [23, 13] that an effective large-distance description

³⁴Understandably, some are rendered prudent by the realization that the ratio between the Planck scale and the energy scales we are probing with modern particle colliders is so big that it is, for example, comparable (within a couple of orders of magnitude) to the ratio between the average Earth-Moon distance and the Bohr radius.

of some aspects of quantum gravity might involve quantum symmetries and noncommutative geometry (while at the Planck scale even more novel geometric structures might be required).

The intuition emerging from these considerations on a novel relationship between measuring apparatus and system and by a Wheeler-Hawking picture of the quantum-gravity vacuum has not yet been implemented in a fully-developed new formalism describing the first stage of partial unification of gravitation and quantum mechanics, but one can use this emerging intuition for rough estimates of certain candidate quantum-gravity effects. Some of the theoretical estimates that I reviewed in the preceding sections, particularly the ones on distance fuzziness, can be seen as examples of this.

Besides the possibility of direct experimental tests (such as some of the ones here reviewed), studies of low-energy effective quantum-gravity models might provide a perspective on quantum gravity that is complementary with respect to the one emerging from approaches based on proposals for a one-step full unification of gravitation and quantum mechanics. On one side of this complementarity there are the attempts to find a low-energy effective quantum gravity which are necessarily driven by intuition based on direct extrapolation from known physical regimes; they are therefore rather close to the phenomenological realm but they are confronted with huge difficulties when trying to incorporate this physical intuition within a completely new formalism. On the other side there are the attempts of one-step full unification of gravitation and quantum mechanics, which usually start from some intuition concerning the appropriate formalism (*e.g.*, canonical/loop quantum gravity or critical superstrings) but are confronted by huge difficulties when trying to “come down” to the level of phenomenological predictions. These complementary perspectives might meet at some mid-way point leading to new insight in quantum gravity physics. One instance in which this mid-way-point meeting has already been successful is provided by the mentioned results reported in Ref. [41], where the candidate phenomenon of quantum-gravity induced deformed dispersion relations, which had been proposed within phenomenological analyses [46, 23, 5] of the type needed for the search of a low-energy theory of quantum gravity, was shown to be consistent with the structure of canonical/loop quantum gravity.

11.2 Theories on non-commutative Minkowski space-time

At various points in these notes there is a more or less explicit reference to deformed symmetries and noncommutative space-times³⁵. Just in the previous subsection I have recalled the conjecture [23, 13] that an effective large-distance description of some aspects of quantum gravity might involve quantum symmetries and noncommutative geometry. The type of *in vacuo* dispersion which can be tested [5] using observations of gamma rays from distant astrophysical sources is naturally encoded within a consistent deformation of Poincaré symmetries [23, 27, 28].

A useful structure (at least for toy-model purposes, but perhaps even more than that) appears to be the noncommutative (so-called “ κ ”) Minkowski space-time [64, 65, 21]

$$[x^i, t] = i\lambda x^i, \quad [x^i, x^j] = 0 \quad (46)$$

where $i, j = 1, 2, 3$ and λ (commonly denoted³⁶ by $1/\kappa$) is a free length scale. This simple noncommutative space-time could be taken as a basis for an effective description of phe-

³⁵The general idea of some form of connection between Planck-scale physics and quantum groups (with their associated noncommutative geometry) is of course not new, see *e.g.*, Refs. [101, 102, 103, 104, 21, 105, 106, 107, 108]. Moreover, some support for noncommutativity of space-time has also been found within measurability analyses [32, 23].

³⁶This author is partly responsible [28] for the redundant convention of using the notation λ when the reader is invited to visualize a length scale and going back to the κ notation when instead it might be natural for the reader to visualize a mass/energy scale. In spite of its unpleasantness, this redundancy is here reiterated in order to allow the reader to quickly identify/interpret corresponding equations in Ref. [28].

nomena associated with a nontrivial foamy quantum-gravity vacuum³⁷. When probed very softly such a space would appear as an ordinary Minkowski space-time³⁸, but probes of sufficiently high energy would be affected by the properties of the quantum-gravity foam and one could attempt to model (at least some aspects of) the corresponding dynamics using a noncommutative Minkowski space-time. In light of this physical motivation it is natural to assume that λ be related to the Planck length.

The so-called κ -deformed Poincaré quantum group [104] acts covariantly [65] on the κ -Minkowski space-time (46). The dispersion relation for massless spin-0 particles

$$\lambda^{-2} (e^{\lambda E} + e^{-\lambda E} - 2) - \vec{k}^2 e^{-\lambda E} = 0, \quad (47)$$

which at low energies describes a deformation that is linearly suppressed by λ (and therefore, if indeed $\lambda \sim L_p$, is of the type discussed in Section 5), emerges [21, 27, 28] as the appropriate Casimir of the κ -deformed Poincaré group. Rigorous support for the interpretation of (47) as a *bona fide* dispersion relation characterizing the propagation of waves in the κ -Minkowski space-time was recently provided in Ref. [28].

In Ref. [28] it was also observed that, using the quantum group Fourier transform which was worked out for our particular algebra in Ref. [110], there might be a rather simple approach to the definition of a field theory on the κ -Minkowski space-time. In fact, through the quantum group Fourier transform it is possible to rewrite structures living on noncommutative space-time as structures living on a classical (but nonAbelian) “energy-momentum” space. If one is content to evaluate everything in energy-momentum space, this observation gives the opportunity to by-pass all problems directly associated with the non-commutativity of space-time. While waiting for a compelling space-time formulation of field theories on noncommutative geometries to emerge, it seems reasonable to restrict all considerations to the energy-momentum space. This approach does not work for *any* noncommutative space-time but only for those where the space-time coordinate algebra is the enveloping algebra of a Lie algebra, with the Lie algebra generators regarded ‘up side down’ as noncommuting coordinates [111].³⁹

Within this viewpoint a field theory is not naturally described in terms of a Lagrangian, but rather it is characterized directly in terms of Feynman diagrams. In principle, according to this proposal a given ordinary field theory can be “deformed” into a counterpart living in a suitable noncommutative space-time not by fancy quantum-group methods but simply by the appropriate modification of the momentum-space Feynman rules to those appropriate for a nonAbelian group. Additional considerations can be found in Ref. [28], but, in order to give at least one example of how this nonAbelian deformation could be applied, let me observe here that the natural propagator of a massless spin-0 particle on κ -Minkowski space-time should be given in energy-momentum space by the inverse of the operator in the dispersion relation (47), i.e. in place of $D = (\omega^2 - \vec{k}^2 - m^2)^{-1}$ one would take

$$D_\lambda = \left(\lambda^{-2} (e^{\lambda \omega} + e^{-\lambda \omega} - 2) - e^{-\lambda \omega} \vec{k}^2 \right)^{-1}. \quad (48)$$

³⁷In particular, within one particular attempt to model space-time foam, the one of *Liouville* non-critical strings [19], the time “coordinate” appears [109] to have properties that might be suggestive of a κ -Minkowski space-time.

³⁸Generalizations would of course be necessary for a description of how the quantum-gravity foam affects spaces which are curved (non-Minkowski) at the classical level, and even for spaces which are Minkowski at the classical level a full quantum gravity of course would predict phenomena which could not be simply encoded in noncommutativity of Minkowski space.

³⁹Another (partly related, but different) κ -Minkowski motivated proposal for field theory was recently put forward in Ref. [112]. I thank J. Lukierski for bringing this paper to my attention.

As discussed in Ref. [28] the elements of this approach to field theory appear to lead naturally to a deformation of CPT symmetries, which would first show up in experiments as a violation of ordinary CPT invariance. The development of realistic field theories of this type might therefore provide us a single workable formalism⁴⁰ in which both *in vacuo* dispersion and violations of ordinary CPT invariance could be computed explicitly (rather than being expressed in terms of unknown parameters), connecting all of the aspects of these candidate quantum-gravity phenomena to the value of $\lambda \equiv 1/\kappa$. One possible “added bonus” of this approach could be associated with the fact that also loop integration must be appropriately deformed, and it appears plausible [28] that (as in other quantum-group based approaches [103]) the deformation might render ultraviolet finite some classes of diagrams which would ordinarily be affected by ultraviolet divergences.

12 CONSERVATIVE MOTIVATION AND OTHER CLOSING REMARKS

Since this paper started off with the conclusions, readers might not be too surprised of the fact that I devote most of the closing remarks to some additional motivation. These remarks had to be postponed until the very end also because in reviewing the experiments it would have been unreasonable to take a conservative viewpoint: those who are so inclined should find in the present lecture notes encouragement for unlimited excitement. However, before closing I must take a step back and emphasize those reasons of interest in this emerging phenomenology which can be shared even by those readers who are approaching all this from a conservative viewpoint.

In reviewing these quantum-gravity experiments I have not concealed my (however moderate) optimism regarding the prospects for data-driven advances in quantum-gravity research. I have reminded the reader of the support one finds in the quantum-gravity literature for the type of phenomena which we can now start to test, particularly distance fuzziness and violations of Lorentz and/or CPT symmetries and I have also emphasized that it is thanks to recent advances in experimental techniques and ideas that these phenomena can be tested (see, for example, the role played by the remarkable sensitivities recently achieved with modern interferometers in the experimental proposal reviewed in Section 4 and the role played by very recent break-throughs in GRB phenomenology in the experimental proposal reviewed in Section 5). But now let me emphasize that even from a conservative viewpoint these experiments are extremely significant, especially those that provide tests of quantum mechanics and tests of fundamental symmetries. One would not ordinarily need to stress this, but since these lectures are primarily addressed to young physics students let me observe that of course this type of tests is crucial for a sound development of our science. Even if there was no theoretical argument casting doubts on them, we could not possibly take for granted (extrapolating *ad infinitum*) ingredients of our understanding of Nature as crucial as its mechanics laws and its symmetry structure. We should test quantum mechanics and fundamental symmetries anyway, we might as well do it along the directions which appear to be favoured by some quantum-gravity ideas.

One important limitation of the present stage in the development of quantum-gravity phenomenology is the fact that most of the experiments actually test only one of the two main

⁴⁰Until now the young field of quantum-gravity phenomenology has relied on “single-use” phenomenological models (the parameters of the phenomenological model are only relevant in one physical context). A first step toward a greater maturity of this phenomenological programme would be the development of phenomenological models that apply to more than one physical context (the same parameters are fitted using data from more than one physical context). The type of field theory on κ -Minkowski space-time that was considered in Ref. [28] (with its single parameter λ) could represent a first example of these more ambitious multi-purpose phenomenological models.

branches of quantum-gravity proposals: the proposals in which (in one or another fashion) quantum decoherence is present. There is in fact a connection (whose careful discussion I postpone to future publications) between decoherence and the type of violations of Lorentz and CPT symmetries and the type of power-law dependence on T_{obs} of distance fuzziness here considered. The portion of our community which finds appealing the arguments supporting the decoherence-inducing Wheeler-Hawking space-time foam (and certain views on the so-called “black-hole information paradox”) can use these recent developments in quantum-gravity phenomenology as an opportunity for direct tests of some of its intuition. The rest of our community has developed an orthogonal intuition concerning the quantum-gravity realm, in which there is no place for quantum decoherence. The fact that we are finally at least at the point of testing decoherence-involving quantum-gravity approaches (something which was also supposed to be impossible) should be seen as encouragement for the hope that even other quantum-gravity approaches will eventually be investigated experimentally.

Even though there is of course no guarantee that this new phenomenology will be able to uncover important elements of the structure of quantum gravity, the fact that such a phenomenological programme exists suffices to make a legitimate (empirical) science of quantum gravity, a subject often derided as a safe heaven for theorists wanting to speculate freely without any risk of being proven wrong by experiments. As emphasized in Refs. [87, 113] (and even in the non-technical press [114]) this can be an important turning point in the development of the field. Concerning the future of quantum-gravity phenomenology let me summarize my expectations in the form of a response to the question posed by the title of these notes: I believe that we are indeed at the dawn of quantum-gravity phenomenology, but the forecasts call for an extremely long and cloudy day with only a few rare moments of sunshine. Especially for those of us motivated by theoretical arguments suggesting that at the end of the road there should be a wonderful revolution of our understanding of Nature (perhaps a revolution of even greater magnitude than the one undergone during the first years of this 20th century), it is crucial to profit fully from the few glimpses of the road ahead which quantum-gravity phenomenology will provide.

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